

Applied Dynamical Systems
LTCC Examination 2010
Examination question

1. Calculate the diffusion coefficient $D(3)$ for the map

$$B_3(x) := \begin{cases} 3x & , \quad 0 \leq x < 1/2 \\ 3x - 2 & , \quad 1/2 \leq x \leq 1 \end{cases} ,$$

continued onto the whole real line by $B_3(x + 1) = B_3(x) + 1$, by using the escape rate formalism described in Section 2.2.3 of the lecture notes. Note that you do not need to reproduce Step 1 of this formalism. Work out Step 2 and 3 for this map along the same lines as demonstrated in the lecture notes, as follows:

- (a) Find a Markov partition for $B_3(x)$ by suitably splitting the unit interval into two partition parts.
- (b) Write down the matrix Frobenius-Perron equation for $B_3(x)$ by constructing the transition matrix $T(3)$.
- (c) Solve the eigenvalue problem of $T(3)$ by writing down the eigenvalue equation for two representative vector components. Then reduce them to one single equation, which can be solved along the same lines as in the lecture notes.
- (d) Proceed by matching the largest eigenmode of $T(3)$ to the largest one of the diffusion equation, in complete analogy to how it was done for $T(4)$ under Step 3 in the lecture notes.