# LTCC Models, exam question 2009–10

1. (a) Recently members of the University of New South Wales proposed a formula for the optimum time to propose (it seems to assume that the man proposes). To work out when to "pop the question" you follow the process that they give:

   - First of all, set out the last possible age by which you want to get married, for example, 39. Call this number $n$.
   - Then, decide the earliest age at which you'll start to consider women as potential wife material, for example, from when you turn 20 onwards. This age becomes $p$.
   - Subtract $p$ from $n$ (i.e $39 - 20$), then multiply the result by 0.368. This gives you 6.992, which then needs to be added back to your minimum age (20), which more or less equals 27.
   - This result is your optimal proposal age. Ideally you should not propose to anyone before you hit this age, but afterwards you should prepare to pop the question to the very next girl you date as long as she's the best of the bunch so far.

   They say the figure of 0.368 comes from the equation

$$\sum_{r=2}^{n-k+1} \frac{(n-k)}{n}\frac{(n-k-1)}{(n-1)}\frac{(n-k-2)}{(n-2)}\cdots\frac{(n-k-r+2)}{(n-r+2)} \times \frac{k}{(n-r+1)} \times \left(\frac{1}{r-1}\right)$$

   Comment on this model.

   See `http://www.maths.unsw.edu.au/news/2010/marriageproblem.pdf`

   (b) (i) Explain how, with a suitable choice of $M$ and $N$, the expansion of Veronis (1968, J. Fluid Mech., **34**, pp. 315–336, equations (2.13) and (2.14)) can yield the set of 5 ordinary differential equations of Da Costa, Knobloch & Weiss (1981) given in lectures.

   (ii) Linearise the Da Costa *et al.* equations, and derive all the points of marginal stability giving $r_T$ as a function of the other parameters.

   (iii) Investigate the steady solutions of the full set of rescaled equations.

   (iv) Given the assumptions made in the analysis, comment on the likely applicability and limitations of applying this analysis to the problem of water between two horizontal boundaries with imposed temperature and salinity gradients. You may assume that $\sigma \approx 7$ and $\tau \approx 1/80$.