

LTCC Analytical Methods Exam 2010

Consider the following partial differential equation for $f(x, y)$ in the domain $-\infty < x < \infty$ and $0 < y < \pi/4$,

$$\frac{\partial}{\partial x} (2xf \tan y) + \frac{1}{\cos y} \frac{\partial}{\partial y} (f \cos^3 y) + 3f \sin y \cos y - \epsilon \nabla^2 f = 0.$$

The boundary conditions to be imposed are

$$f(x, 0) = 2x \quad \text{and} \quad f(x, \pi/4) = 1.$$

- (a) Find the general solution to the governing equation with $\epsilon = 0$ and explain whether the boundary conditions can be satisfied.
- (b) Solve the problem when $0 < \epsilon \ll 1$.
- (c) How would your solution change if the upper boundary condition was

$$f(x, \pi/4) - 2\epsilon \frac{\partial f}{\partial z}(x, \pi/4) = 0$$

instead, with the additional requirement that $f(x, \pi/4)$ is non-singular for all x ? Determine the solution in this case.