

**LTCC Mock Examination Question 2010: Algebraic Topology**

1. State theorems which describe the fundamental groups a) of a circle  $S^1$ , b) of a 2-sphere  $S^2$ , and give a careful outline proof of one of them.

2. The space  $X$  is formed as follows.  $U, V$  are disjoint discs on a 2-sphere;  $X_0$  is the sphere with the interiors of  $U, V$  removed.  $X_1$  is a standard cylinder ( $S^1 \times I$ ). Show that the boundaries of  $X_0$  and  $X_1$  are a pair of circles.

Glue  $X_0$  to  $X_1$  by a homeomorphism of the boundaries — call the result  $X$ . By varying the homeomorphism of boundaries, how many different (non-homeomorphic) possibilities are there for  $X$ ? In each case, by constructing a map from  $X$  onto  $S^1$  (or otherwise), prove that  $\pi_1(X)$  is infinite.