

LTCC Models, exam question 2008–09

1. (a) According to Dr Len Fisher of Bristol University, the following formula can be used to calculate the correct amount of gravy to make to go with a meal:

$$G = 100 \left(\frac{W - (D/S)}{D} \right)$$

where W = uncooked weight of the food, D = the cooked weight of the food, S = the shrinkage factor, G = the gravy absorption index. An example from the accompanying publicity is that of a 20g potato. This weighs 16g after roasting and shrinks in volume by 7%, giving $S = 0.93$ and will have a gravy index of $G = 20.833$. Thus it will need 20.833% of its cooked weight in gravy. This formula can then be applied to all the elements on a plate to give the optimum amount of gravy, and so avoid wastage. Comment on this formula's appropriateness for calculating your gravy need. Can it be improved?

- (b) A biological model for the the interaction of life and nutrients in the ocean is given by the following model based on nitrogen concentration:

$$\frac{dN}{dt} = -uIP \frac{N}{(K_s + N)} + eD + (1 - \alpha)gPZ$$

$$\frac{dP}{dt} = uIP \frac{N}{(K_s + N)} - gPZ - sP$$

$$\frac{dZ}{dt} = \alpha gPZ - dZ$$

$$\frac{dD}{dt} = dZ + sP - eD$$

where N , P , Z and D represent the nitrogen concentration in the available nutrients, the phytoplankton (photosynthesising plankton), the zooplankton (which eat the phytoplankton and each other), and the detritus (solid waste products which can break down to release nutrients). With the conservation of nitrogen we have

$$N_c = N + P + Z + D = \text{constant.}$$

The term I is the average intensity of the sunlight that drives the photosynthesis. Apart from N , P , Z and D all other terms are taken to constant.

- Interpret the terms in these equations, and explain if possible why they take these forms.
- Are there alternatives to any of these terms?
- It is possible to find analytic expressions for 4 steady states of this system. Two are particularly tedious to find, but the other two are relatively straightforward if one looks solutions of certain forms based on realistic assumptions about the possible population. Find these two relatively simple solutions, and comment on the conditions for their existence.
- What important features are missing from this model?