

# Statistical Modelling and Estimation

## Exam Question

February 20, 2009

The following data on a response variable  $Y$  were collected from an experiment with explanatory variables  $X_1$ ,  $X_2$  and  $X_3$ .

$X_1$	$X_2$	$X_3$	$Y$
-1	-1	-1	12.3
-1	-1	1	9.7
-1	1	-1	14.3
-1	1	1	13.9
1	-1	-1	24.6
1	-1	1	25.7
1	1	-1	22.2
1	1	1	28.1
0	0	0	19.4
0	0	0	21.5
0	0	0	20.1
0	0	0	19.7

The proposed model is

$$E(Y_i) = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_{11} 3x_{1i}^2 + \beta_{22} x_{2i}^2 + \beta_{33} x_{3i}^2 + \beta_{12} x_{1i} x_{2i} + \beta_{13} x_{1i} x_{3i} + \beta_{23} x_{2i} x_{3i},$$

with  $Y_i$ s uncorrelated and  $V(Y_i) = \sigma^2$ .

1. Express the model in matrix form, i.e. write down the design matrix  $\mathbf{X}$  and the vector of parameters  $\boldsymbol{\beta}$ .
2. Find the rank of the matrix  $\mathbf{X}$ .
3. Find a generalized inverse of  $\mathbf{X}'\mathbf{X}$ .

4. Hence find a least squares estimator  $\hat{\beta}$  of  $\beta$ .
5. Show that  $\beta_{11}$  is not estimable.
6. Show that  $\theta = \beta_{11} + \beta_{22} + \beta_{33}$  is estimable and find the form of its least squares estimator  $\hat{\theta}$  and its observed value.
7. Find the variance of  $\hat{\theta}$ .
8. Write down the form of a  $100(1 - \alpha)\%$  confidence interval for  $\theta$ .