

# LTCC

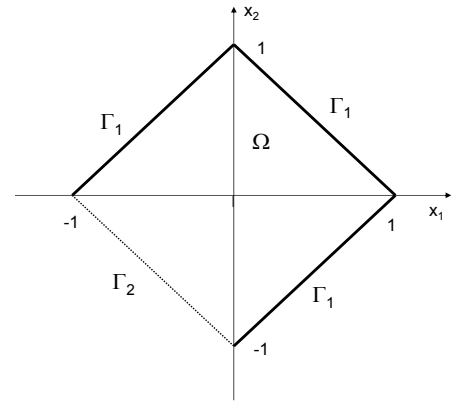
## Applied Computational Methods

### Exam Question 2008-2009

Let  $\Omega$  be a diamond with the diagonal length 2, centered at point  $(0,0)$ , with the boundary  $\Gamma = \bar{\Gamma}_1 \cap \bar{\Gamma}_2$ , see Figure. Consider the boundary value problem

$$\begin{aligned} -\operatorname{div} \left[ (2 + x_1^2 + x_2^2) \nabla u(x) \right] + 2x_2^2 u(x) &= f(x) \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \Gamma_1, \\ \frac{\partial u}{\partial n} &= g \quad \text{on } \Gamma_2, \end{aligned}$$

where  $f \in L_2(\Omega)$ ,  $g \in L_2(\Gamma_2)$ ,  $x = (x_1, x_2) \in \Omega$ .



1. Derive a variational formulation for this problem.
2. Prove that the variational formulation has a unique solution in an appropriate space.
3. Give the general finite-element formulation of the problem.
4. Calculate a finite element piece-wise linear approximation of the solution on the uniform triangular mesh consisting of two triangular elements and then calculate  $u(0, 1/2)$  for  $f(x) = 2$ ,  $g(x) = 1$  and make a conclusion about accuracy of the 2-element approximation.