

# LTCC Advanced Course

- Title: Syzygies and homological algebra
- Lecturer Details:
  - Lecturer: Professor F E A Johnson
  - Lecturer home institution: University College London
  - Lecturer e-mail: feaj [at math.ucl.ac.uk]
- Basic Details:
  - Core Audience: 2nd/3rd year: pure
  - Course Format: extended (10 hours at 2 hours per week)
  - Timing: late autumn 2009
- Course Description:
  - Overview:

The primary aim of this course is to give an introduction to modern homological algebra from the standpoint of syzygies and the derived module category.

The treatment is motivated by applications in nonsimply connected homotopy theory, especially the  $D(2)$  problem; that is, the homotopy classification of group presentations.
  - Topics:
    - 1) Schanuel's Lemma and stable modules.
    - 2) Syzygies and the derived module category.
    - 3) Resolutions and cohomology. Co-representability of module cohomology.
    - 4) Algebraic  $n$ -complexes and their homotopy classification.
    - 5) Group presentations, Cayley complexes and the  $D(2)$ -problem.

The treatment will be illustrated throughout by examples, primarily from the cohomology theory of discrete groups.
  - Recommended reading:
    - i) 'Stable modules and the  $D(2)$  problem', F.E.A.Johnson, CUP 2003
    - ii) 'Homology', S. MacLane, Springer 1963
  - Prerequisites: There is no single absolutely necessary prerequisite for the course. However it would be highly desirable to have seen some sample (co)-homology calculations (particularly useful would be cohomology of groups); to be happy with the idea of working with modules over fairly arbitrary rings; and to have some idea of both representation theory of finite groups and also of group presentations.