Topological Arakelov Theory and nonabelian cohomology

The title 'Topological Arakelov Theory' is a convenient thread by which one may bind the algebraic theory of group extensions to the geometric theory of smooth fibre bundles. In Arakelov Theory (as the term is understood in Arithmetic/Algebraic Geometry) one considers varieties $X$ which fibre over a complex curve $B$ with fibre a quasi-projective variety $F$; in the first instance the fibre is also a curve, possibly punctured. Such a fibration gives rise to an exact sequence of groups

$$1 \to \pi_1(F) \to \pi_1(X) \to \pi_1(B) \to 1.$$ 

Elementary examples show that, perhaps contrary to expectations, the topology of $X$ is not completely determined by the group extension (even in the most favourable case where all spaces are aspherical). One of our aims will be to attempt to measure what is lost on passing from geometry to algebra.

Provisional Timetable

**Lecture 1**: Group extension theory and the fibration theory of simplicial sets.

**Lecture 2**: Realising group extensions by fibering Hilbert manifolds. The theorems of Eells-Elworthy and Chapman-Wong.

**Lecture 3**: Fibre reduction: realising group extensions by fibering finite dimensional manifolds.

**Lecture 4**: Finiteness theorems for poly-Fuchsian groups.

**Lecture 5**: Smoothing theorems for poly-Fuchsian groups. The theorems of Gramain, Eells and Earle.

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