LTCC Advanced Course

Title: Combinatorial Optimisation: Packing, Partitioning, and Covering

Basic Details:

- Core Audience: All Ph.D. students in Pure and Applied Mathematics
- Course Format: 5x2hr lectures

Course Description:

Packing, partitioning, and covering are basic problems that lie at the heart of almost every problem in Combinatorial Optimisation. In this course, we study important examples of such problems in Graph Theory, namely, vertex colouring, maximum matching, and multicommodity flows. This leads us to the theories of perfect and weakly bipartite graphs. We also cover the structural, polyhedral, and algorithmic aspects of such graphs.

Tentative Outline:

Lecture 1: Minimum vertex colouring, Tutte-Berge formula for maximum matchings, Edmonds' Blossom Algorithm, postman sets in graphs
Lecture 2: Chromatic number, maximum cliques, perfect graphs, Meyniel graphs
Lecture 3: Multicommodity flows, the cut condition, weakly bipartite graphs
Lecture 4: Blocking and anti-blocking polyhedra, perfect matrices, ideal matrices
Lecture 5: Separation vs. optimisation in linear programming, semidefinite programming, algorithms for perfect and weakly bipartite graphs

Recommended reading:

1. Cornuéjols. Combinatorial Optimization: Packing and Covering. SIAM (2001)
2. Cook, Cunningham, Pulleyblank, Schrijver. Combinatorial Optimization. Wiley (1997)
3. Abdi. Lecture notes in Packing and Covering. Available here (2018)
4. Lecture notes for the 2020 edition of this course which can be found here. (I plan to cover more this year.)

Prerequisites: Basic knowledge of graph theory, linear programming, and algorithms

Format:

- Five sets of electronic lecture notes.
- Five sheets of exercises appended to the end of each set of lecture notes.

Lecturer Details:

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