LTCC Advanced Course

Title: Introduction to Random Topology

Basic Details:

- Core Audience: All students
- Course Format: Extended 5 x 2hr lectures

Course Description:

- **Keywords:**
  random graphs, random topology, algebraic topology, simplicial complexes

- **Syllabus:**
  Random graphs are rich mathematical objects that have numerous applications (e.g., in network science, biology, machine learning). Bringing together probability theory, combinatorics, and algebraic topology, in this module we will introduce the theory of random simplicial complexes. These are high-dimensional generalisations of random graphs that in addition to vertices and edges may also contain triangles, tetrahedra and higher-order simplexes. We will start by reviewing the fundamental properties of random graphs and discuss how these properties can be generalised to higher-dimensions using algebraic topology. This will be done mainly using the language of homology (algebraic topology), which we will introduce as well. We will present the main probabilistic theorems for random simplicial complexes, describing phase transitions and central limit theorems related to high-dimensional connectivity, cycles, and more.

  A tentative outline:
  - Lecture 1: Introduction and some basic probabilistic tools
  - Lecture 2: Random graphs: vertex degrees, connectivity and cycles
  - Lecture 3: Introduction to simplicial homology/cohomology
  - Lecture 4: The random d-complex: homological connectivity and d-acyclicity
  - Lecture 5: The random clique complex

- **Recommended reading:**
  - Introduction to Random Graphs, Frieze and Karoński
  - Topology of random simplicial complexes: a survey, Kahle
  - Random Simplicial Complexes: Models and Phenomena, Bobrowski and Krioukov

- **Additional Optional reading:**
  Will be given during the lectures.

- **Prerequisites:**
  Good understanding of the foundations of probability theory and linear algebra (at

an undergraduate level). Familiarity with basic concepts in graph theory will be beneficial.

Format:

- No of discussion/problem sheets: 4
- Electronic lecture notes: Will be provided after each lesson.

Lecturer Details:

- Lecturer: Dr. Omer Bobrowski
- Lecturer home institution: Queen Mary University of London
- Lecturer e-mail: o.bobrowski@qmul.ac.uk