LTCC Advanced Course

Title: Introduction to Spectral Geometry

Basic Details:

- Core Audience: 2nd and 3rd year, Pure, 1st year students also encouraged to attend
- Course Format (**Extended**: 5 x 2hr lectures)

Course Description:

- Keywords: Spectral geometry, Geometric analysis, Shape optimisation, Laplace eigenvalues, Minimal Surfaces, Heat Trace

- Syllabus: Spectral geometers study the relationship between the geometry of an object and the eigenvalues/eigenfunctions of elliptic operators (such as the Laplacian) acting on them. It is an area of mathematics at the interface of many other fields: Partial differential equations, measure theory, geometric analysis, probability theory and even analytic number theory. In this course we will focus on two major modern aspects of the theory:
    - Which geometric objects arise as shape optimisers for the the first few eigenvalues? Minimal surfaces are a notoriously elusive object in geometric analysis, and we will see how critical points for the eigenvalues naturally give rise to minimal surfaces in spheres.
    - What is the distribution of eigenvalues at infinity, and what geometric information can be extracted from this distribution? In general, we will be able to see coarse information like the volume, or the area of the boundary. In dimension 2, we may even recover topological type. These questions will be answered using a variational viewpoint, using the modern language of "measure eigenvalues" and "admissible measures", in particular using methods from the recent works of Laugesen, Freitas–Laugesen, Girouard–Karpukhin–Lagacé and Karpukhin–Stern

- Recommended reading: There are not too many modern books on the subject, so the recommended reading is mostly a book in progress and some set of lecture notes.
    - Michael Levitin, Dan Mangoubi and Iosif Polterovich, *Topics in Spectral Geometry*, *https://michaellevitin.net/Book/*
    - Yaiza Canzani, *Analysis on manifolds via the Laplacian*, *https://canzani.web.unc.edu/wp-content/uploads/sites/12623/2016/08/Laplacian.pdf*
- Additional Optional reading:
    - Rupert L. Frank, Ari Laptev and Timo Weidl, *Schrödinger operators: eigenvalues and Lieb-Thirring inequalities* Cambridge studies in Advanced Mathematics, 2022.
    - Additional optional readings will be announced at a later time.

- Prerequisites: Some familiarity with Riemannian geometry, measure theory and functional analysis.

Format:

- No of discussion/problem sheets: 4 problem sheets
- Electronic lecture notes: provided at the beginning of the course

Lecturer Details:

- Lecturer: Jean Lagacé
- Lecturer home institution: King's College London
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