

Measure Theory: Exercises 3

1. Show that there exists a Borel set $A \subseteq \mathbf{R}$ such that $0 < \lambda(I \cap A) < \lambda(I)$ for every open interval I of finite length.
2. Show that if a set $B \subseteq \mathbf{R}$ satisfies $\lambda^*(B) > 0$ then B includes a set that is not Lebesgue measurable.
3. Let μ and ν be finite measures on some measurable space (X, \mathcal{A}) . Show that \mathcal{A}_μ and \mathcal{A}_ν need not be equal. Prove or disprove: $\mathcal{A}_\mu = \mathcal{A}_\nu$ if and only if μ and ν have exactly the same sets of measure zero.
4. Let (X, \mathcal{A}, μ) be a measure space. Show that for every subset A of X that $\mu^*(A) + \mu_*(X \setminus A) = \mu(X)$.
5. Let (X, \mathcal{A}, μ) be a measure space. Show that if $E_i \subseteq A \subseteq F_i$ and $\mu(F_i \setminus E_i) = 0$ for $i = 1, 2$ then $\mu(E_1) = \mu(E_2) = \mu(F_1) = \mu(F_2)$.