

LTCC Basic Course

Title: Measure-Theoretic Probability

Basic Details:

- Core Audience: Students with interest in Probability and its applications to Statistics, Analysis, Physics and other fields.
- Course format: extended, 10h (5x2h lectures)

Course Description:

The course provides a measure-theoretic background for the modern Probability Theory, including convergence and limit theorems, and introduces into basic types of random processes used in applications

- Keywords: Measure theory, sigma-field, random variable, filtration, stochastic process, martingale, diffusion, convergence of random variables and processes, Brownian motion, Poisson process, diffusion, Gaussian process, Itô calculus.
- Syllabus: Measure-theoretic background of Probability Theory, random variable as measurable function, expectation as integral, filtration of sigma-algebras as information flow, modes of convergence and limit theorems. Conditional probability and expectation via the Radon-Nikodym theorem, martingales and the basic martingale convergence theorems. Stochastic processes with discrete and continuous time, general Markov processes, diffusions, Lévy processes, Gaussian processes and fields. Brownian motion, path properties, Donsker's invariance principle, elements of Itô calculus.
- Prerequisites: A second course in Probability, given usually under the names Probability II, Applied Probability, Stochastic Modelling or similar. The LTCC course "Measure Theory" (Basic) is advised as preparation for this course.
- Course Text: David Williams, *Probability with Martingales*, CUP, 1991.
- Additional/Optional Reading:
 - O. Kallenberg, *Foundations of Modern Probability*, 2nd ed., Springer 2002.
 - G. R. Grimmett and D. Stirzaker, *Probability and Random Processes*, 3rd ed., OUP, 2001 [ch. 12 and 13]
 - J. M. Steele, *Stochastic Calculus and Financial Applications*, Springer 2001 [ch.1-6].Further references will be provided in the lectures.

Format:

- No. of discussion/problem sheets: 4
- Lecture notes: printed lecture notes and other materials will be posted on the course website.
- Lecture/Computer session split: 10/0h

Lecturer Details:

- Lecturer: Professor Alexander Gnedin
- Lecturer's home institution: School of Mathematical Sciences, Queen Mary, University of London