LTCC "MORSE THEORY, TOPOLOGY AND ROBOTICS" EXAM 2022 - 2023

EXAMINER: PROFESSOR MICHAEL FARBER

1.

- (a) Give the definition of a vector field on a manifold,
- (b) What is meant by a 1-parameter group of diffeomorphisms,
- (c) State the theorem about 1-parameter group of diffeomorphisms generated by a vector field.

2.

- (a) Give the definition of a Morse critical point;
- (b) State the Morse Lemma;
- (c) State the theorem about changes in the sub-level set when crossing a non-degenerate critical level.
- (d) State the Morse inequalities.
- (e) What is the minimal number of critical points of a Morse function on a closed orientable surface of genus g?

3.

- (a) Describe a Morse function on \mathbb{CP}^n and find all its critical points.
- (b) Give the definition of Lusternik Schnirelmann category.
- (c) State the category of the sphere S^2 , torus T^2 , surface of genus 2?

4.

- (a) Give the definition of a linkage;
- (b) Describe the configuration space of a planar linkage.
- (c) What is meant by the length vector of a linkage?
- (d) When do we say that the length vector is generic?
- (e) State the theorem about Betti numbers of moduli spaces of planar linkages.