Proposal: LTCC Basic course

Title: Orthogonal Polynomials and Special Functions

Basic Details:

Core audience: This is a 10h course intended for Students working in applied analysis with interests in: integrable systems, mathematical physics, spectral theory of operators, approximation theory, random matrix theory as well as some aspects of computational and numerical methods.

Course format: Extended.

Course Description:

Keywords: orthogonal polynomials, special functions, moment problem, recurrence relations, Jacobi operators.

Syllabus

The course aims to introduce the main ideas and methods of the modern theory of orthogonal polynomials, covering the following topics, together with examples of a variety of applications (including number theory, random matrices and probability):

- Brief introduction to special functions: from Gamma function to the (generalised) hypergeometric functions, their characterisations from different viewpoints.
- General theory of orthogonal polynomials (OPs): definition of orthogonal polynomials including some basic notions from the theory of measure and integration; determinant representation and the Hankel determinants, existence and uniqueness; recurrence relations and the Christoffel-Darboux identities; study of algebraic, analytic and probabilistic properties of the zeros of an orthogonal polynomial sequence; OPs and the spectral analysis of linear operators including Jacobi operators; the Stieltjes transform and continued fractions as well as Padé approximation.
- Special families of orthogonal polynomials: the classical polynomials of Hermite, Laguerre, Jacobi and Bessel; semiclassical polynomials and the connection with Painlevé equations; families of discrete OPs (of classical and semiclassical type).
- A brief introduction to the multiple orthogonal polynomials and discussion on some examples and applications.

References

[1] Beals, R. and R . Wong: Special Functions, Cambridge University Press, Cambridge, UK, 2010.

[2] T. S. Chihara: An Introduction to Orthogonal Polynomials, Gordon and Breach, 1978, reprinted Dover, 2011.

[3] M. Domínguez de la Iglesia, Orthogonal Polynomials in the Spectral Analysis of Markov Processes: Birth-Death Models and Diffusion. Encyclopedia of Mathematics and Its Applications. Cambridge: Cambridge University Press, 2021.

[4] M. E. H. Ismail: Classical and Quantum Orthogonal Polynomials in One Variable, Cambridge University Press, 2005.

[5] A. Martínez-Finkelshtein and W.Van Assche , "WHAT IS... A Multiple Orthogonal Polynomial?", Notices of the American Mathematical Society (2016), 63(9), 1029-1031.

[6] G. Szegő: Ortogonal polynomials, Amer. Math. Soc., Fourth ed., 1975.

[7] W. Van Assche, Encyclopedia of Special Functions: The Askey-Bateman Project. Edited by M. E. H. Ismail. Vol. 1. Cambridge: Cambridge University Press, 2020.

[8] W. Van Assche, Orthogonal Polynomials and Painlevé Equations. Australian Mathematical Society Lecture Series. Cambridge: Cambridge University Press, 2017.

[9] (online resources) NIST Digital Library of Mathematical Functions.

Format

- Electronic lecture notes
- Necessary support facilities: none
- Necessary software requirements for computing facilities: None
- Lecture/tutorial or discussion hour split: 8/2. Part of each 2-hour session will be set aside to discuss issues arising from the current lecture or from the problems and/or reading set the previous week.

Lecturer Details

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