LTCC Advanced Course

Title: *Probabilistic Analysis of Algorithms*

Basic Details:

- Core Audience (1st yr or 2nd/3rd yr: pure, app. or stats): *App, 1st- 3rd yr.*
- Course Format (**Extended**: 5 x 2hr lectures)

Course Description:

- Keywords:

contraction methods, divide-and-conquer algorithms, linear random recurrences, probability metrics, Quicksort, random trees.

- Syllabus:

The classical analysis of algorithms is mainly concerned either with average or with extremal performance properties of computer algorithms. Assuming a random input, one usually asks, what is the average performance of an algorithm, or what is the worst (the best) performance? The probabilistic analysis of algorithms is a modern field lying in the intersection of probability theory and computer science that is used to analyze more delicate performance features of algorithms. Its methods and techniques allow one to answer questions like: what is the probability that the working time of an algorithm does not exceed a prescribed threshold? what is the confidence interval for the working time of an algorithm of a prescribed significance level? The aim of the course is to explain how to formulate and answer the above questions and why they are useful. During the course a student will become acquainted both with mathematical machinery used in the field, as well as with various applications, including probabilistic analysis of sorting algorithms, search algorithms, graph algorithms and general divide-and-conquer algorithms.

Lecture 1: The classic Quicksort Algorithm: where has the probabilistic analysis of algorithms started from?
Lecture 2: Required mathematical toolbox: spaces of probability measures and probability metrics. The Banach fixed-point theorem for probability measures.
Lecture 3: Divide-and-conquer paradigm. Random linear recurrences and their examples: variations of Quicksort and random trees.
Lecture 4: The contraction method for linear random recurrences with non-degenerate limit equations and examples.
Lecture 5: The contraction method for linear random recurrences with degenerate limit equations and examples.

- Recommended reading:

1. *Alsmeyer G. (2012). Random recursive equations and their distributional fixed points.*
https://www.uni-muenster.de/Stochastik/lehre/WS1112/StochRekGleichungenII/book.pdf
2. *Neininger R. (2010). Probabilistic Analysis of Algorithms.*
https://dmg.tuwien.ac.at/nfn/neininger/survey.pdf
3. *Marynych A. (2023). Probabilistic Analysis of Algorithms.*

https://do.csc.knu.ua/marynych/wp-content/uploads/sites/2/2023/01/PAA-EN.pdf

4. *Greene D. H., Knuth D.E. (1990). Mathematics for the analysis of algorithms, Birkhauser, 120 p.*

- Additional Optional reading:

1. *Neininger R. and Rueschendorf L. (2004). On the contraction method with degenerate limit equation. Ann. Probab. 32, pp. 2838–2856.*
2. *Roesler U. (1991). A limit theorem for "Quicksort". RAIRO, Inform. Theor. Appl. 25, pp. 85–100.*
3. *Roesler U. (2001). On the analysis of stochastic divide and conquer algorithms. Algorithmica 29, p. 238–261.*
4. *Zolotarev, V. M. (1997). Modern theory of summation of random variables, VSP, Utrecht, The Netherlands, p. 347.*

- Prerequisites:

Probability II, Functional Analysis, Complex Analysis.

Format:

- No of discussion/problem sheets (typically 4 for with solutions):

4 problem sheets with solutions (3-4 exercises per sheet). The solutions will be provided in due course.

- Electronic lecture notes (these are strongly encouraged, as they will form the core of the individual study of the students):

Extended lecture notes are available here:
https://do.csc.knu.ua/marynych/wp-content/uploads/sites/2/2023/01/PAA-EN.pdf
This LTCC course is based mostly on Chapter 4 of the above lecture notes. For the LTCC course it is advisable to start with Introduction and proceed immediately to Chapter 4 consulting with Chapters 1-3 when necessary.

- Necessary support facilities:

A beamer and a blackboard.

Lecturer Details:

- Lecturer: Prof. Alexander Marynych
- Lecturer home institution: Taras Shevchenko National University of Kyiv, Leverhulme Visiting Professor at Queen Mary University of London
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