

Problem Set 1

Problem 1: Let $\{e_1, e_2, e_3\}$ be a \mathbb{Z} -basis for N . Let $\sigma = \text{span}_{\mathbb{R}_{\geq 0}} \{e_1, e_2, e_3\}$. Identify U_σ and U_τ for each face τ of σ . Now consider the cone complex of these faces τ . Does this cone complex define a toric variety? If so, can you identify it? If not, why not?

(This is a teaser for the next lecture.)

Problem 2: Show that U_σ is normal.

(Hint: Observe that S_σ is a saturated subsemigroup of M , and use the lemma.)

Problem 3: Problem 2 suggests a way to construct some non-normal toric varieties. Can you give such a construction?

Problem 4: Show that the cuspidal cubic plane curve $X = \text{Spec}(\mathbb{C}[x, y] / \langle x^3 - y^2 \rangle)$ is non-normal. If you did Problem 3, try to use your construction to describe X .