

# LTCC Proposed Course

- Title: Complex analysis
- Basic Details:
  - Core Audience: First year, pure and applied
  - Course Format: Extended (10 hours at 2 hours per week)
- Course Description:
  - Keywords: Nevanlinna theory, monodromy, Riemann-Hilbert problems, elliptic functions, hypergeometric functions, Painlevé equations.
  - Syllabus:
    - Approximate breakdown by hour:  
  
Part I: The value distribution of meromorphic functions (Nevanlinna theory)
      1. General introduction. The Nevanlinna characteristic. Nevanlinna's first main theorem.
      2. Basic identities and properties of the characteristic function.
      3. The lemma on the logarithmic derivative and applications. Value distribution of meromorphic solutions of ordinary differential equations.
      4. Nevanlinna's second main theorem. Picard's theorem.
      5. Defect relations. Discussion of Vojta's dictionary and connections with Diophantine approximation (additional handouts will be available for students interested in this topic).  
Part II: Special functions in the complex domain
      6. Elliptic functions.
      7. Ordinary differential equations in the complex domain. Existence and uniqueness. Regular singular points. The hypergeometric equation.
      8. The monodromy group. Monodromy for the hypergeometric equation.
      9. Riemann-Hilbert problems.
      10. Applications of Riemann-Hilbert problems. The Painlevé equations.
  - Recommended reading:
    - W. Cherry and Z. Ye, *Nevanlinna's theory of value distribution*, Springer, 2001
    - A. S. Fokas, A. R. Its, A. A. Kapaev and V. Yu. Novokshenov, *Painlevé transcendents; the Riemann-Hilbert approach*, AMS, 2006
    - W. K. Hayman, *Meromorphic functions*, Clarendon Press, 1964
    - E. L. Ince, *Ordinary differential equations*, Dover, 1956
  - Additional Optional reading:
    - E. Bombieri and W. Gubler, *Heights in Diophantine geometry*, CUP, 2006
    - F. D. Gakhov, *Boundary value problems*, Pergamon Press, 1966
    - A. A. Goldberg and I. V. Ostrovskii, *Value distribution of meromorphic functions*, AMS, 2008
    - E. Hille, *Ordinary differential equations in the complex domain*, Wiley, 1976
    - Min Ru, *Nevanlinna theory and its relation to Diophantine approximation*, World Scientific, 2001
    - N. I. Muskhelishvili, *Singular integral equations*, 1953
  - Prerequisites: Any standard introductory course on complex analysis.

- Format:
  - No of discussion/problem sheets: 4
  - Electronic lecture notes: Yes.
  - Necessary support facilities: None
  - Necessary software requirements for computing facilities: None
  - Proposed timing: Spring
  - Lecture/computer session/tutorial/discussion split (hours of each): 10 hours of lectures
- Lecturer Details:
  - Lecturer: Professor Rod Halburd
  - Lecturer home institution: University College London
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  - Lecturer telephone number: 020 7679 2973