LTCC Basic Course

Title: Elliptic Operators and the Index Theorem I

Basic Details:
- Core Audience (1^{st}yr : pure):
- Course Format (extended):

Course Description:
- **Keywords:** Manifold, (Co)tangent bundle, Differential Form, Vector Bundle, Differential Operator, Dirac Operator.

- **Syllabus:**
  1. Differential operators (on manifolds): Dirac operators and Clifford modules, de Rham and Dolbeault complexes. (4 hours)

  2. Elliptic differential operators, pseudodifferential operators, parametrices, the Fredholm package for elliptic operators on compact manifolds. (4 hours)

  3. Applications: analytic index of an elliptic operator, continuous dependence on symbol. Hodge theory. (2 hours)

These topics form a self-contained introduction to the theory of elliptic operators and pseudodifferential operators in a geometric context. They also form the background to Part II, which will give the heat-kernel proof of the Atiyah—Singer Index Theorem.

- **Recommended reading:** Friedlander and Joshi for an introduction to the theory of distributions and the Fourier transform; Roe's book `Elliptic operators, topology and asymptotic methods'.

- **Additional optional reading:** Lecture notes of Richard Melrose, http://www-math.mit.edu/~rbm/Lecture_notes.html, the books of Taylor or Shubin on Pseudodifferential operators.,

- **Prerequisites:** I shall assume knowledge of manifolds up to differential forms and the de Rham complex and some familiarity with the language of vector bundles. I also assume you know what a riemannian metric is, and what the metric (aka Levi-Civita) connection is. I shall try to develop the analysis from scratch, but will assume you know what a Hilbert space is. A nodding acquaintance with the Fourier transform will also make the course more digestible.

Lecturer details
- Lecturer: Professor Michael Singer
- Lecturer home institution: UCL