

LTCC Advanced Course

Title: Elliptic Operators and the Index Theorem II

Basic Details:

- Core Audience (1st yr: pure):
- Course Format (extended):

Course Description:

Keywords: Dirac Operator, symbol calculus, Atiyah-Singer index formula, heat kernel, superconnections, characteristic classes.

Syllabus:

1. Traces on pseudodifferential operators. Use of symbol calculus for classical elliptic operators to construct the asymptotic expansion of the heat trace. Identification of the constant term (in t) as the index density. Superconnections and the heat trace. (3 hours)
2. Characteristic classes and genera. Associating multiplicative sequences with specific elliptic operators (Atiyah Singer, Dolbeault, de Rham etc). (2 hours)
3. Proof of the Atiyah-Singer index density formula (3 hours).
4. As time permits: applications of the index formula, outline of index theory for families of operators and determinant line bundles (2 hours).

Recommended reading: Berline, Getzler, Vergne “Heat Kernels and Dirac Operators”; Roe, “Elliptic operators, topology and asymptotic methods”. Various background notes in pdf form will be provided on a course webpage.

Additional background on pseudodifferential operator calculus: in the books of Shubin, Treves and Hormander on Pseudodifferential operators,

Prerequisites: I shall assume knowledge of Part I of this course, though some topics will be very briefly reviewed in order to contextualize what follows. Though, the course will still be usefully intelligible to anyone with some basic background in differential geometry and algebraic topology.

Lecturer Details

Lecturer: Professor Simon Scott, KCL