LTCC PROPOSED COURSE

- **Title**: Introduction to Geometric Analysis on Manifolds

- **Basic Details:**
  -Core Audience: (2nd/3rd yr pure math)
  -Course Format: 5x2hr lectures

- **Course Description:**
  -Keywords: Laplace operator, heat kernel, Green's function, harmonic functions

- **Syllabus**:
  The goal of this course is to provide an introduction to linear elliptic and parabolic theory on complete noncompact Riemannian manifolds. Assuming bounds from below on the Ricci curvature, we will obtain estimates for the Laplace and the heat equations, that are applied to understand the behavior of the heat kernel on such manifolds.

  This theory has important applications to the geometry and topology of manifolds, some of which will be presented here. For example, on a manifold with more than one end we can naturally construct a space of harmonic functions with remarkable properties. In turn, estimating the dimension of that space through geometric considerations helps us understand the topology at infinity of the manifold.

  Below is a detailed plan for the lectures.

  1. The Laplace operator on a complete noncompact manifold. Harmonic functions and Yau's Liouville theorem. The heat equation and Li-Yau heat kernel estimates.
  2. Sobolev, Logarithmic Sobolev inequalities, and their relation to heat kernel estimates.
  3. Long time behavior of the heat kernel. Green's function estimates and solvability of the Poisson equation.
  4. Harmonic functions and the number of ends of a manifold.
  5. Splitting results for manifolds with more than one end. Other applications.

- **Recommended reading:**

  1. Geometric Analysis, by Peter Li, Cambridge Studies in Advanced Mathematics, ISBN-13: 978-1107020641
  2. Heat kernel and analysis on manifolds, by Alexander Grigor'yan, AMS Studies in Advanced Mathematics, ISBN-13: 978-0821893937

- **Prerequisites:**
  Basic knowledge of Riemannian geometry will be assumed. The student is expected to be familiar with standard concepts such as curvature, geodesics, the Bochner formula, and typical calculations using moving frames.
  The course will require some standard tools from the theory of partial differential equations, such as the Sobolev embedding theorem, the fundamental solutions of Laplace and heat equations, and the maximum principle, all in the Euclidean space.

- **Format:**
  Online lecture notes will be available before the start of the course. There will be four problem sheets, to be turned in at the end of each lecture, except in the first week.

- **Lecturer details:**
  Ovidiu Munteanu is an Associate Professor in the Department of Mathematics at University of Connecticut. Should you have any questions, please do not hesitate to write at ovidiu.munteanu@uconn.edu.