LTCC Advanced Pure Course

- Title: **Invariant Theory of Finite Groups**
- Basic Details:
    - Core Audience: Pure
    - Course Format: Advanced (10h)

- Course Description:

  Invariant theory is essentially the study of symmetries of polynomial functions. The symmetries are encoded by an action of a group G on a ring of functions R (by ring automorphisms). The elements of R fixed pointwise by the action form a subring R^G, the ring of invariants. This course will focus on the case of a finite G acting by degree preserving automorphisms on the ring of polynomials R=F [x_1, x_2, …, x_n] with F a field. In this case the homogenous polynomials of degree one can be interpreted as linear functions on a representation V (an FG-module) and the invariant polynomials represent functions on the orbit space V/G. For an algebraically closed field and finite G, the orbit space V/G is a categorical quotient and R^G is the ring of regular functions on this affine variety. We will investigate how properties of the representation V are related to properties of R^G. We will also consider computational questions such as the construction of generating sets for R^G.

- Syllabus:
  1. Introductory examples, motivation and notation.
  2. Finite generation, Noether normalisation, primary and secondary invariants, the Cohen-Macaulay property, the transfer homomorphism, comparing the modular and non-modular cases.
  3. Noether numbers and degree bounds, permutation groups and G\"obel's degree bound, Hilbert series and Molien's theorem.
  4. Reflection groups including theorems of Chevalley, Shephard-Todd, Serre, Nakajima and Kemper-Malle.
  5. Modular invariant theory including the Dickens invariants, modular invariants of finite classical groups, modular invariants of p-groups, depth, cohomological methods, unique factorisation, and the Karaguezian-Symonds decomposition.
  6. Computational methods including SAGBI bases.
  7. Links with algebraic geometry, algebraic topology and group cohomology.

- Format: Proposed timing: early Autumn 2009

- Lecture details: Peter Fleischmann and Jim Shank, Kent.