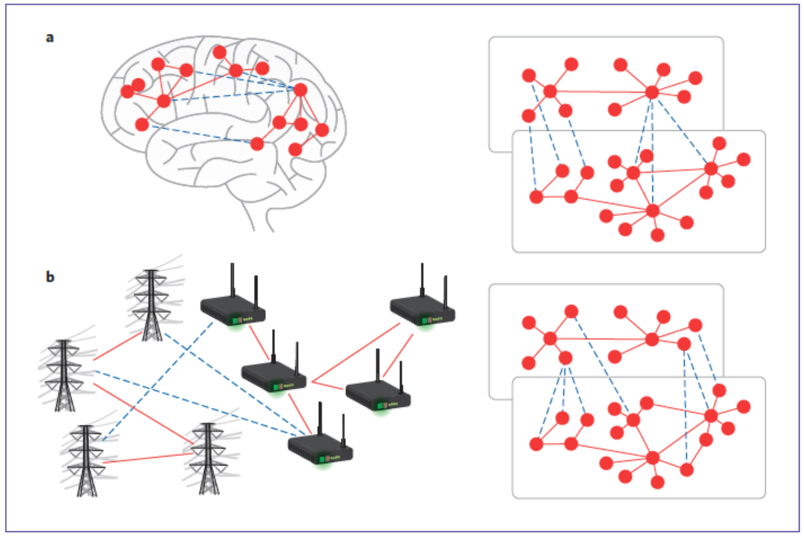

Multilayer Networks:
An introduction

LTCC Course Multilayer Networks
23-24 November 2016

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University of London

Overview of the course

Day 1

- Introduction to multilayer networks and applications
- Structure of multilayer networks
- Generative models

Day 2

- Robustness of multilayer networks
- Message passing equations for percolation in multilayer networks
- Diffusion, epidemic spreading and centrality measures

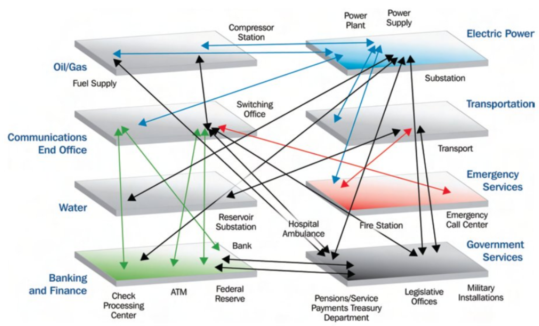
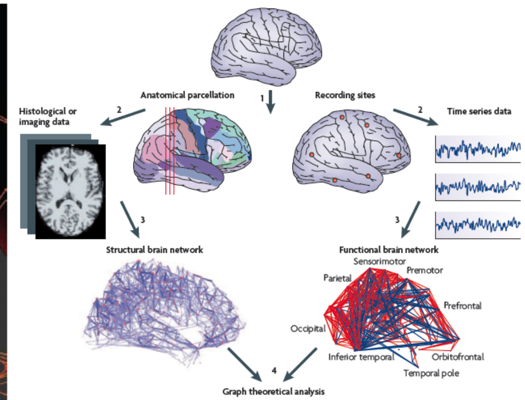


FIGURE 3.1 Connections and interdependencies across the economy. Schematic showing the interconnected infrastructures and their qualitative dependencies and interdependencies. SOURCE: Department of Homeland Security, National Infrastructure Protection Plan, available at http://www.dhs.gov/xprevprot/programs/editorial_0827.shtm.



The function of many complex

technological social and biological systems

depends on the non-trivial interactions between

different networks

Interacting Transportation networks

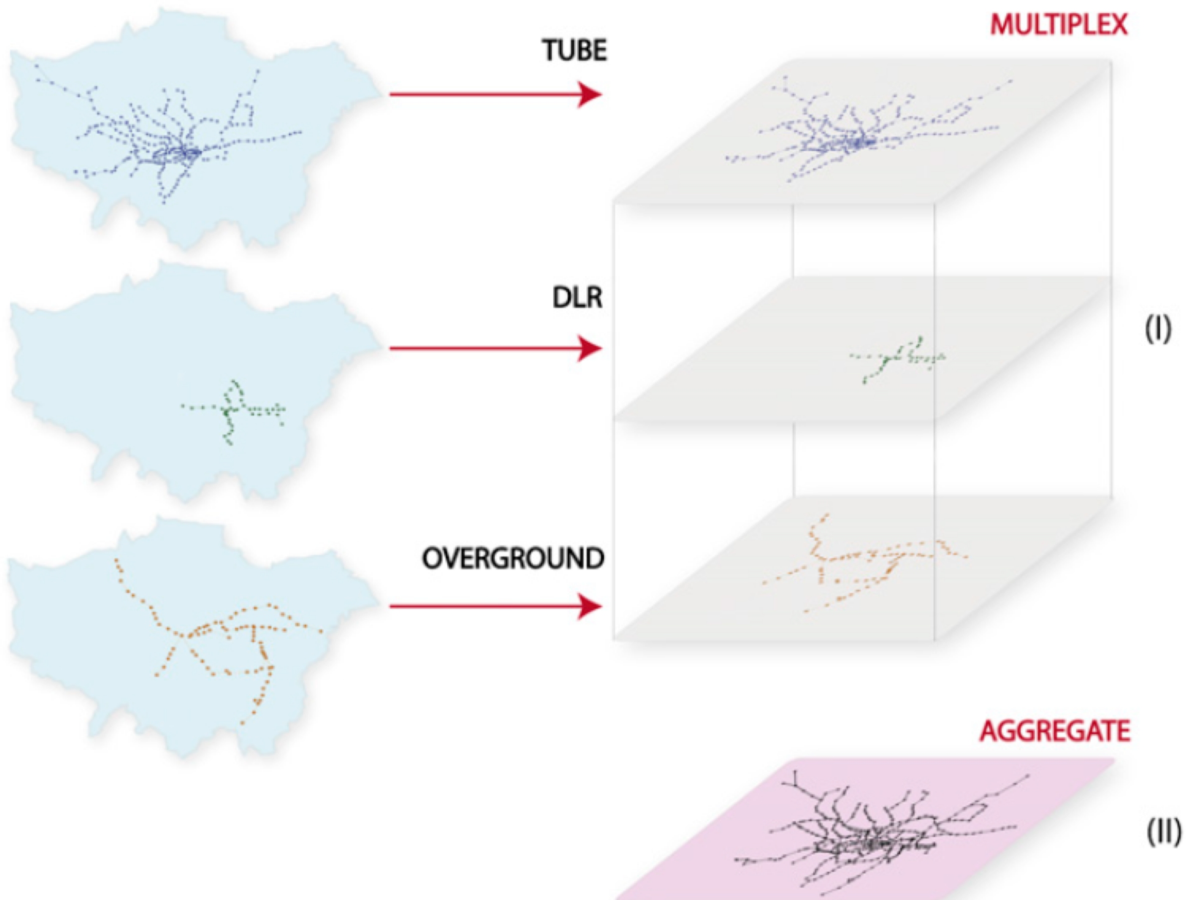
Transportation networks are a major example of interacting networks.

Here
blue lines represent short-range commuting flow by car or train
the red lines indicate airline flow for few selected cities

B. GONÇALVES ET AL., INDIANA UNIV.

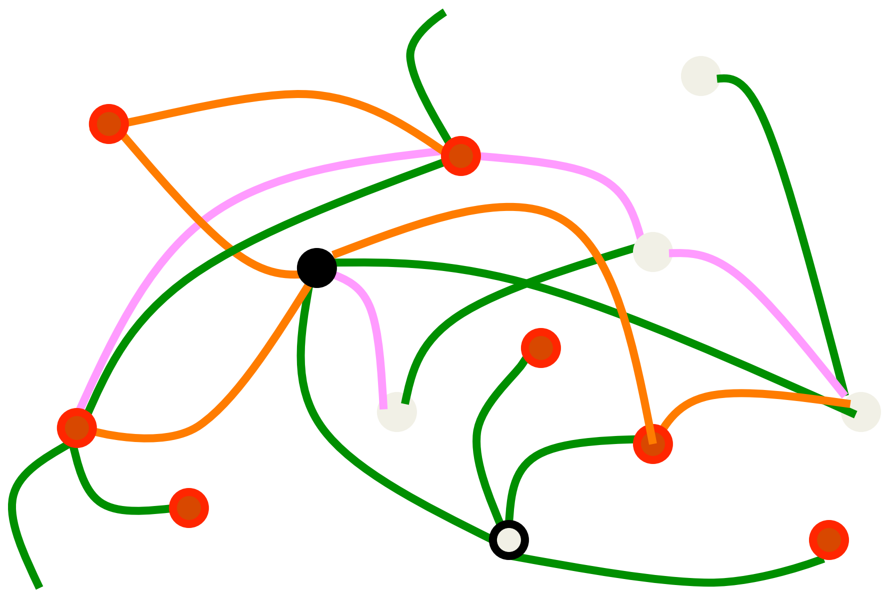
Vespignani Nature 2010

Transportation networks

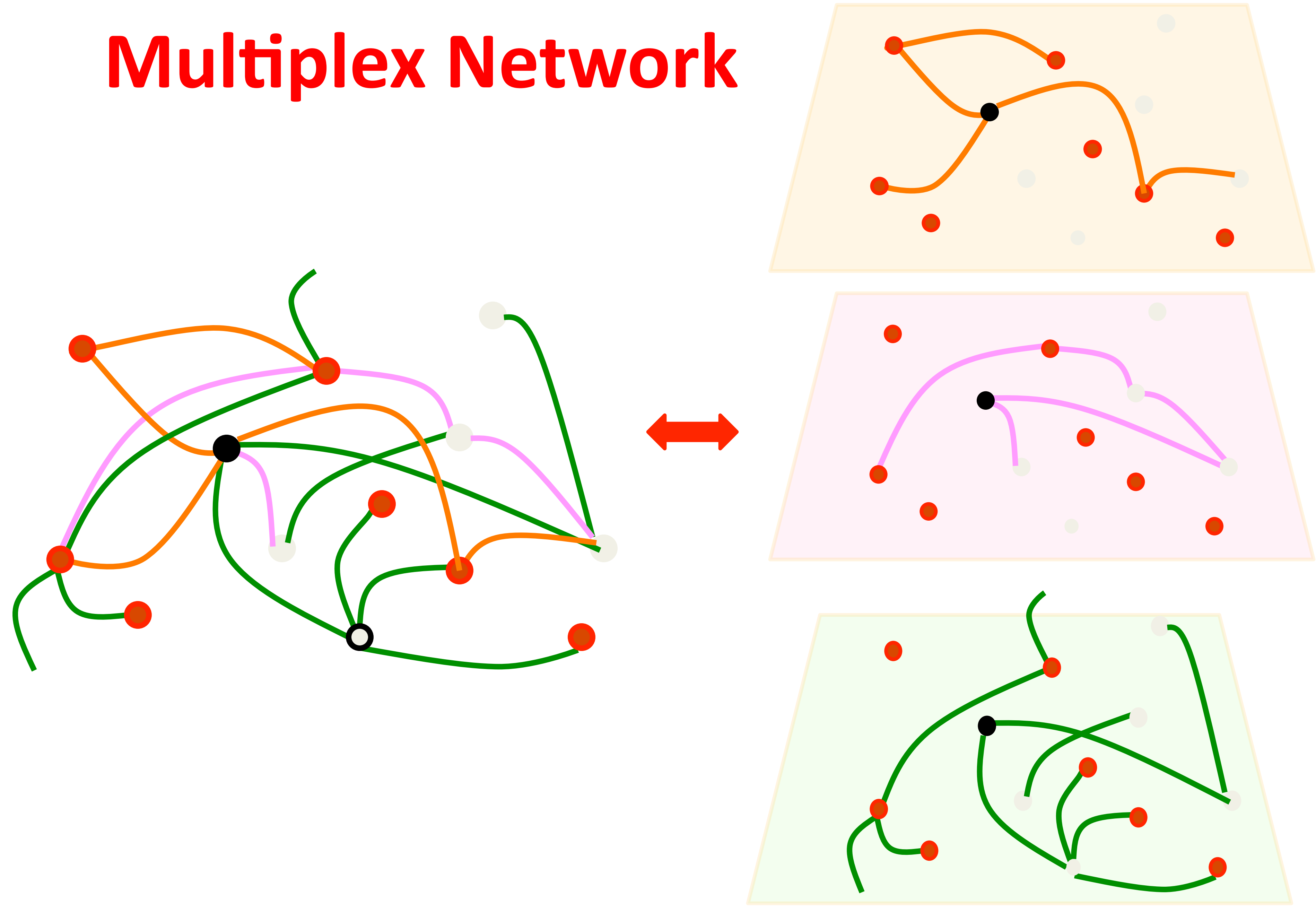


M. De Domenico, A. Solé-Ribalta, S. Gómez, and A. Arenas PNAS (2014)

Multiplex Network

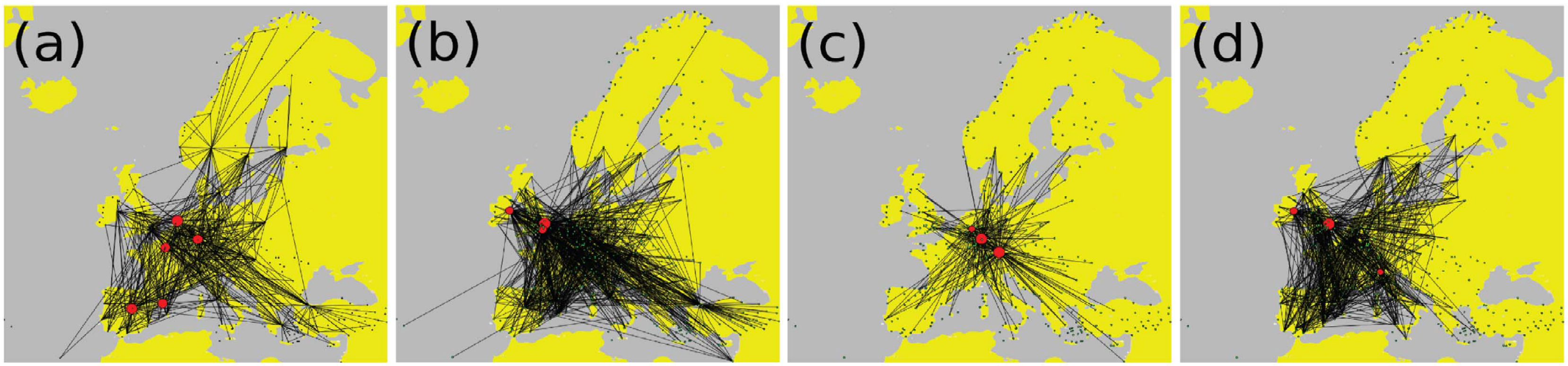


Multiplex Network



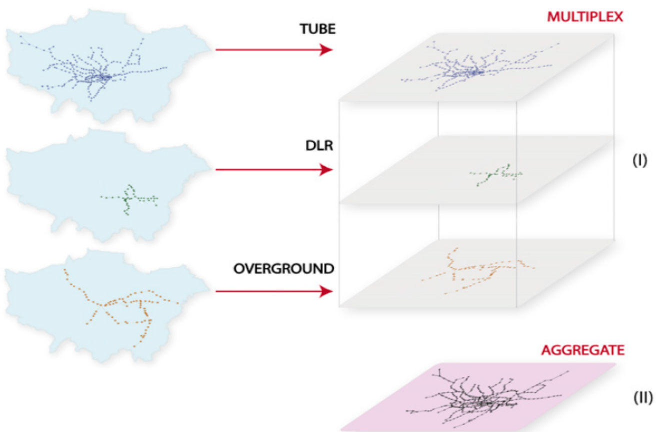
Airline multiplex network

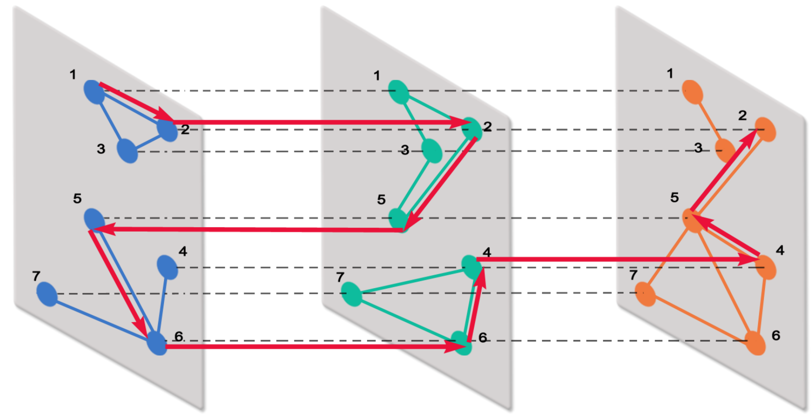
Every airline company is a layer of
the multiplex network



Cardillo et al. Scientific Reports (2013).

Diffusion in multiplex networks





Interlinks are essential for diffusion across the layers of multiplex networks

For high diffusion constant,

diffusion is faster than on the slowest layer, and eventually can yield superdiffusion

S. Gomez et al., Phys. Rev. Lett.(2013)
Radicchi & Arenas, Nature Physics (2013)
M. De Domenico PNAS (2014)

Interconnected infrastructures

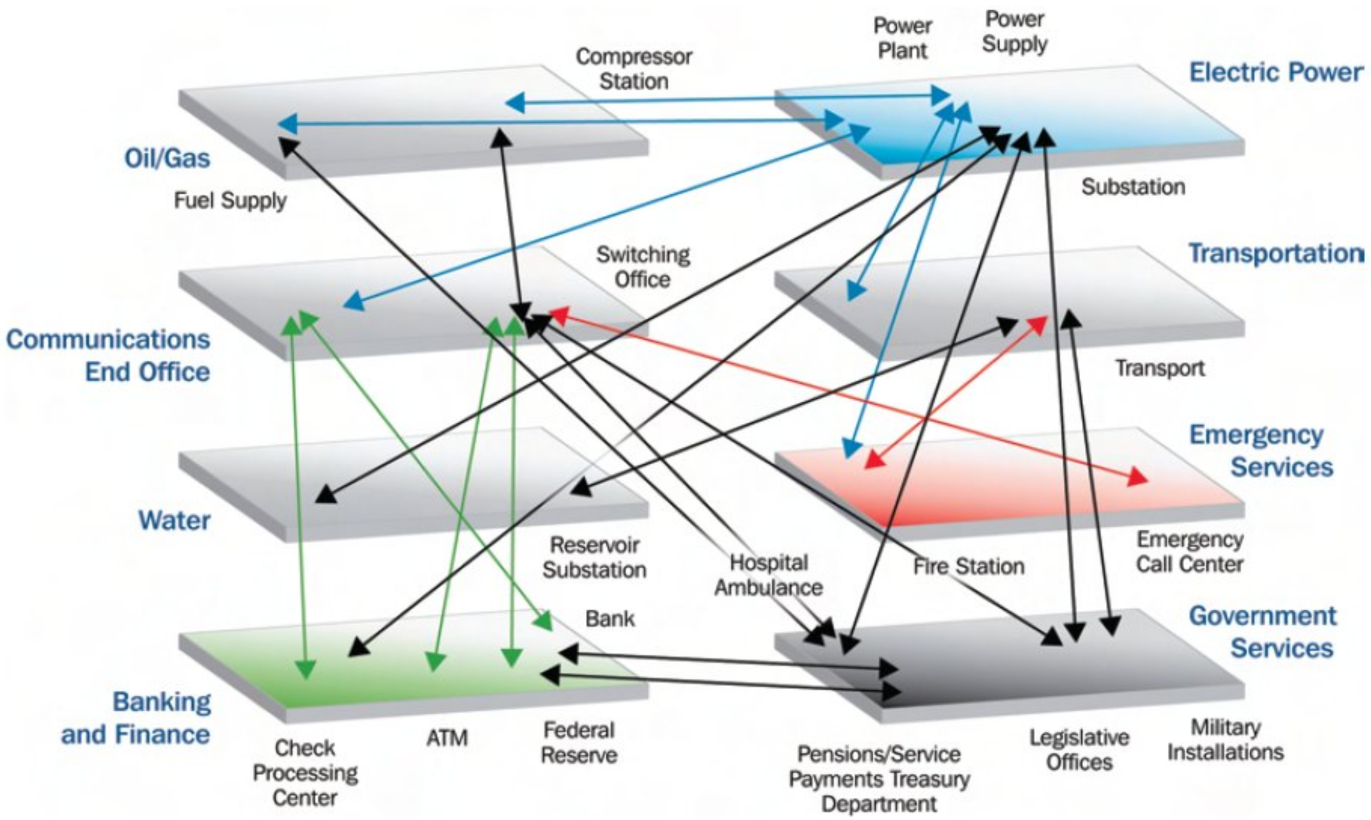
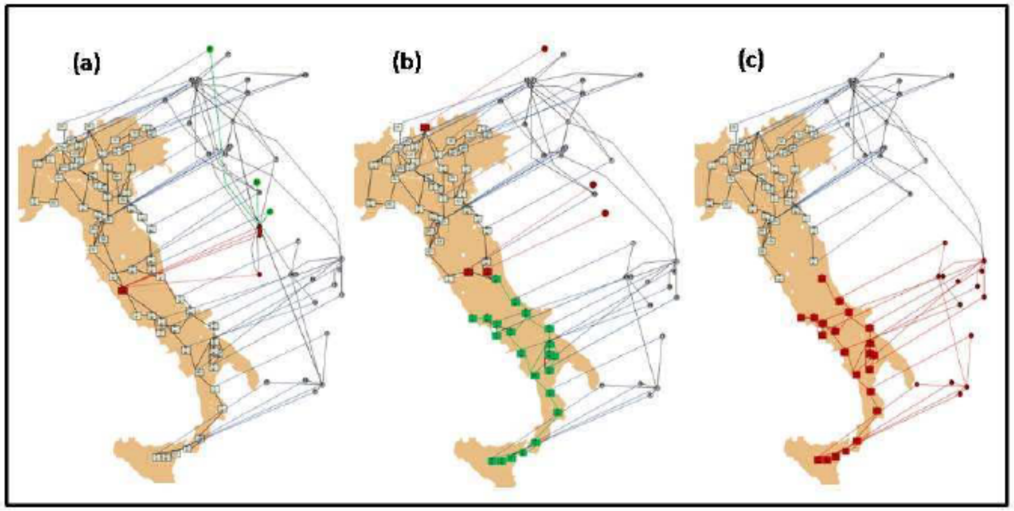


FIGURE 3.1 Connections and interdependencies across the economy. Schematic showing the interconnected infrastructures and their qualitative dependencies and interdependencies. SOURCE: Department of Homeland Security, National Infrastructure Protection Plan, available at http://www.dhs.gov/xprevprot/programs/editorial_0827.shtm.

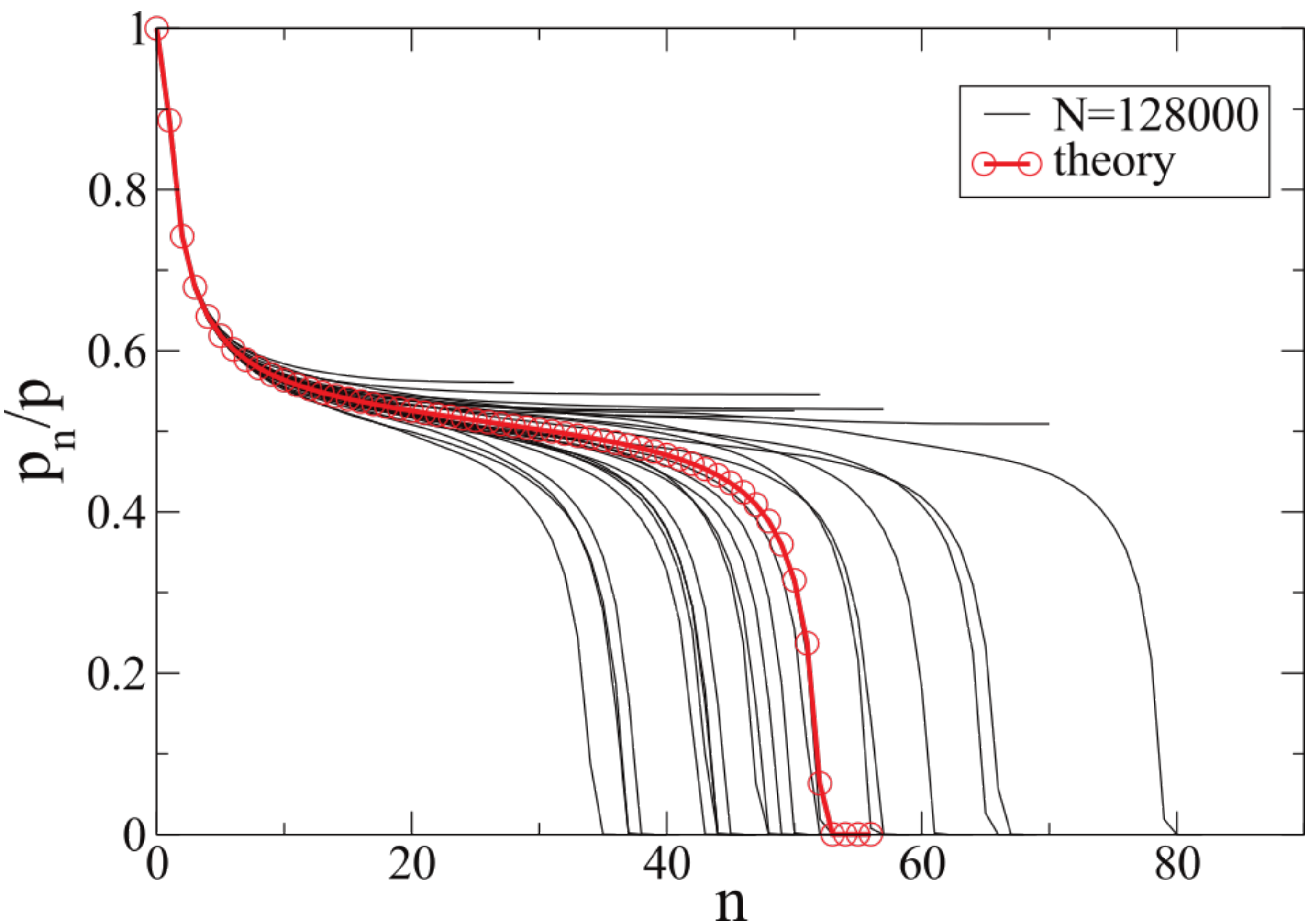
Interdependent infrastructure networks

Complex infrastructures are interdependent and a failure in one network can generate a cascade of failures in the interdependent networks



Buldyrev et al. Nature 2010

Cascade of failure events at the percolation transition



Buldyrev et al. Nature

Nature Physics News & Views

news & views

MULTILAYER NETWORKS

Dangerous liaisons?

Many networks interact with one another by forming multilayer networks, but these structures can lead to large cascading failures. The secret that guarantees the robustness of multilayer networks seems to be in their correlations.

Ginestra Bianconi

Natural complex systems evolve according to chance and necessity — trial and error — because they are driven by biological evolution. The expectation is that networks describing natural complex systems, such as the brain and biological networks within the cell, should be robust to random failure. Otherwise, they would have not survived under evolutionary pressure. But many natural networks do not live in isolation; instead they interact with one another to form multilayer networks — and evidence is mounting that random networks of networks are acutely susceptible to failure. Writing in *Nature Physics*, Saulo Reis and colleagues[1] have now identified the key correlations responsible for maintaining robustness within these multilayer networks.

In the past fifteen years, network theory[2,3] has granted solid ground to the expectation that natural networks resist failure. It has also extended the realm of robust systems to man-made self-organized networks that do not obey a centralized design, such as the Internet or the World Wide Web. In fact, it has been shown that many isolated complex biological, technological and social networks are scale free, meaning that their nodes

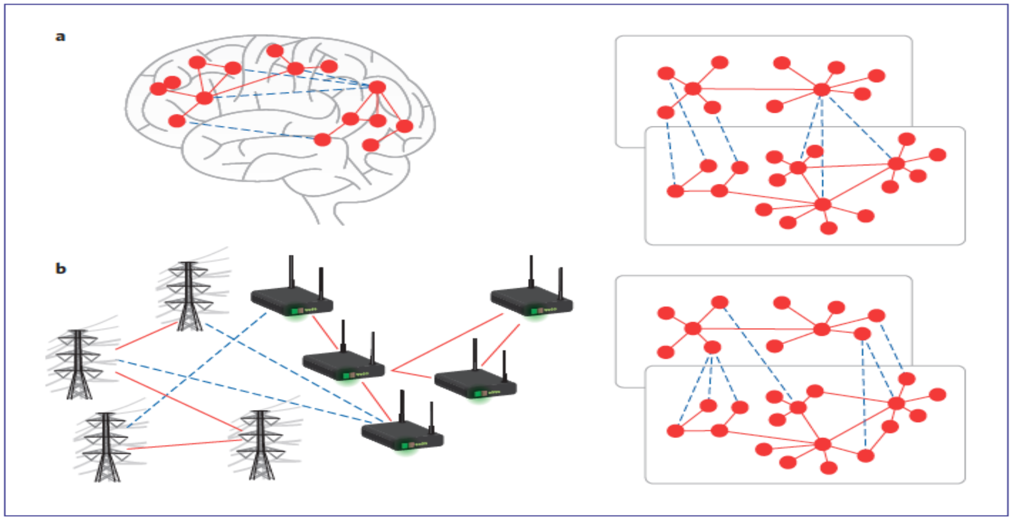


Figure 1 | Reis *et al.*[1] have shown that correlations between intra- (red) and interlayer (blue dotted) interactions influence the robustness of multilayer networks. **a**, In the brain, each network layer has multilayer assortativity and the hubs in each layer are also the nodes with more interlinks, so liaisons between layers are trustworthy. **b**, In complex infrastructures (such as power grids and the Internet), if the interlinks are random, the resulting multilayer network is affected by large cascades of failures[6], and liaisons can be considered dangerous.

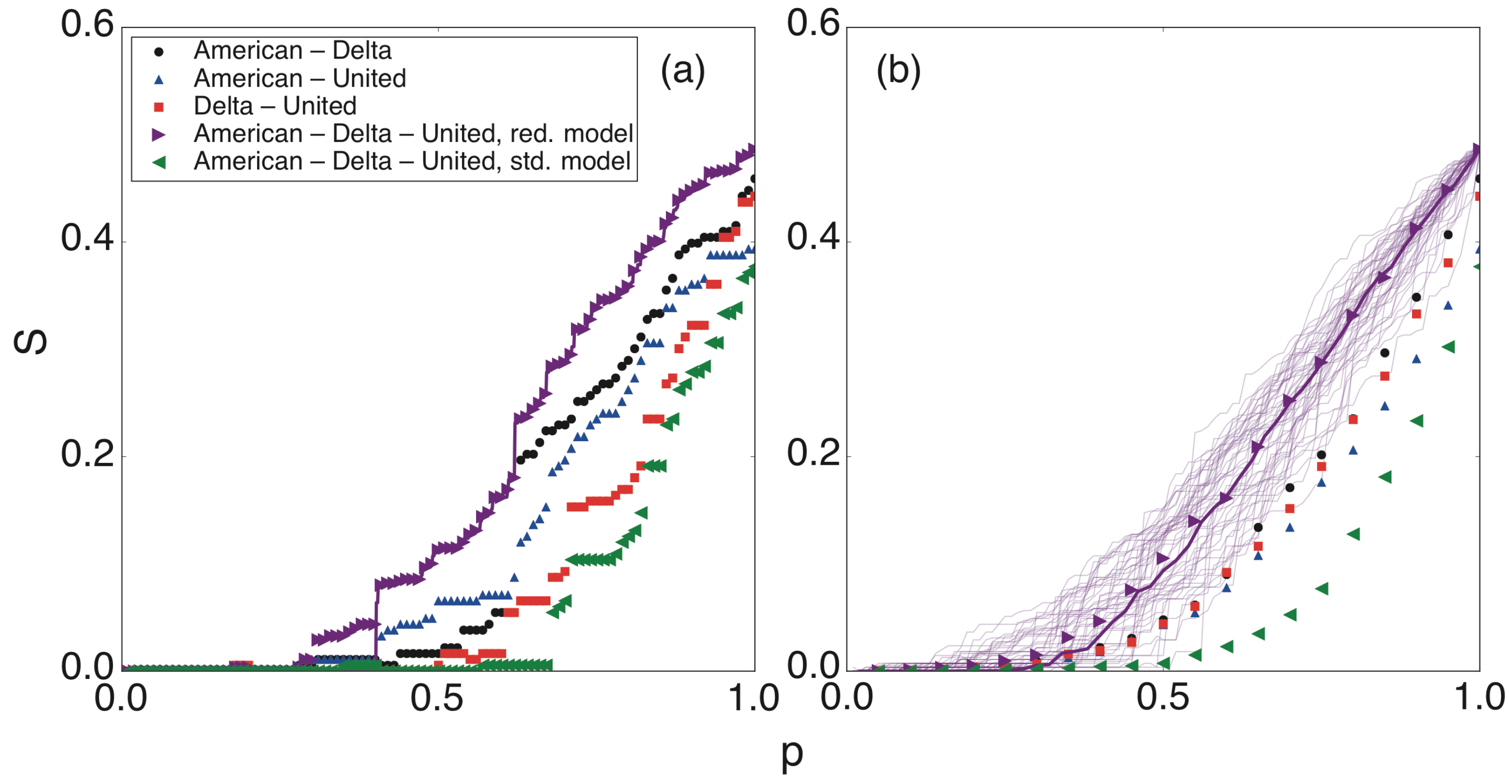
Redundant interdependencies

If a node is interdependent on each one of its replica nodes, the more layers we add the more fragile is a network.

If, instead interdependencies are redundant and a node is the Redundant MCGC if at least one replica node is active,

then more layers we add to the network the more robust it becomes

Redundant interdependencies boost the robustness of multilayer networks



Radicchi Bianconi 2016

Multilayer networks encode more information than single layers

Multilayer networks are not equivalent to a larger single network

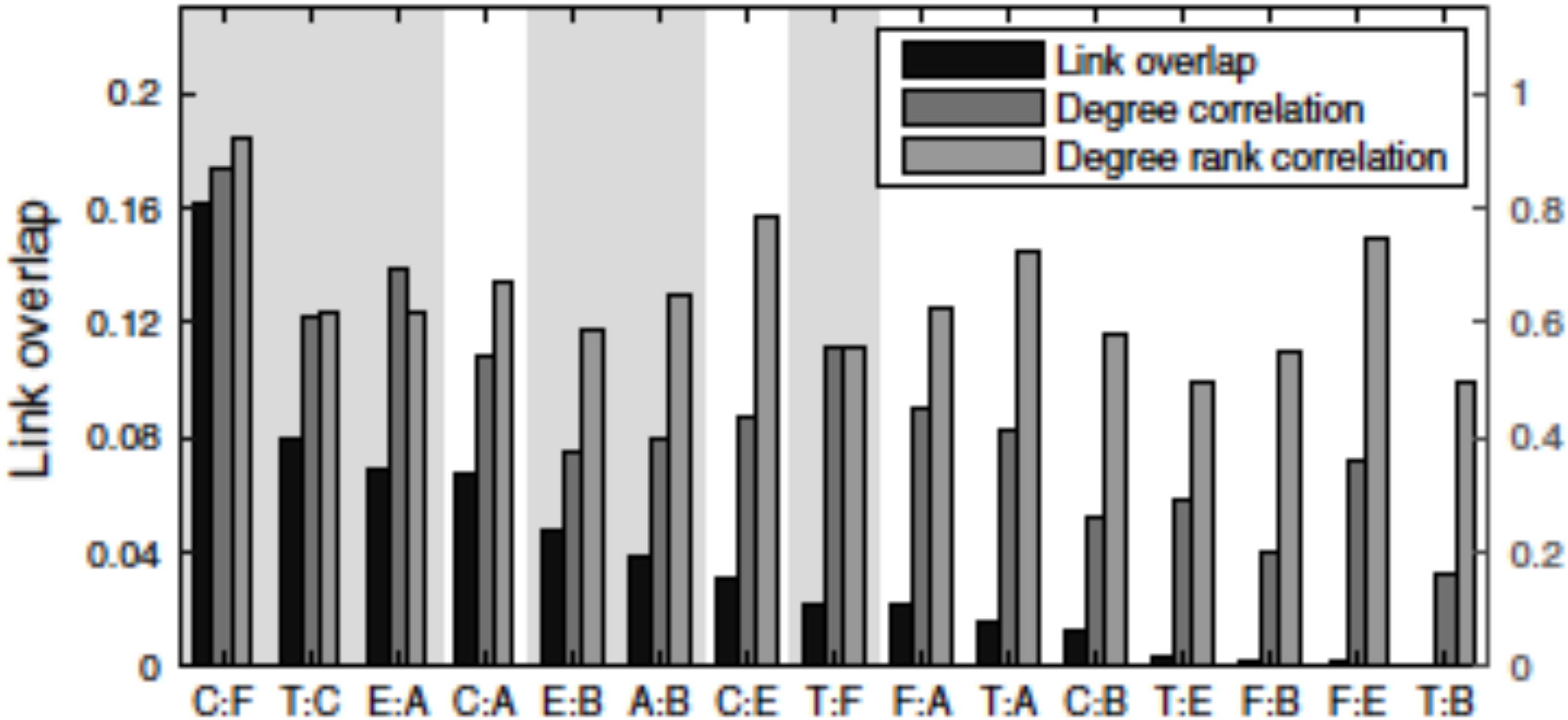
Different types of links
describe different types of interactions,
therefore multilayer networks
encode more information than
their single layers
taken in isolation

Interacting
Social networks

Social networks are interacting and overlapping with profound implications for community detection algorithms

Y.Y. Ahn et al. Nature 2010

Multiplex social networks



Social network of online social game

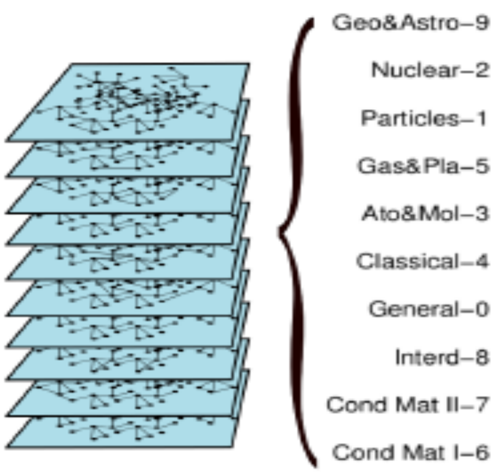
Szell et al . PNAS 2010

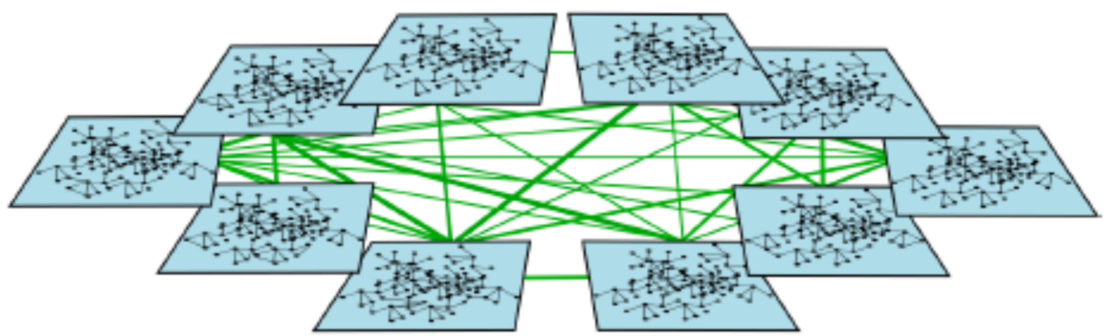
Multiplex community structure of the APS collaboration network

APS collaboration network → 180,538 authors of the American Physical Society papers till 2014 with less than 10 authors

First layer PACS hierarchy (10 layers) → each layer describes the collaboration network in a general field of physics

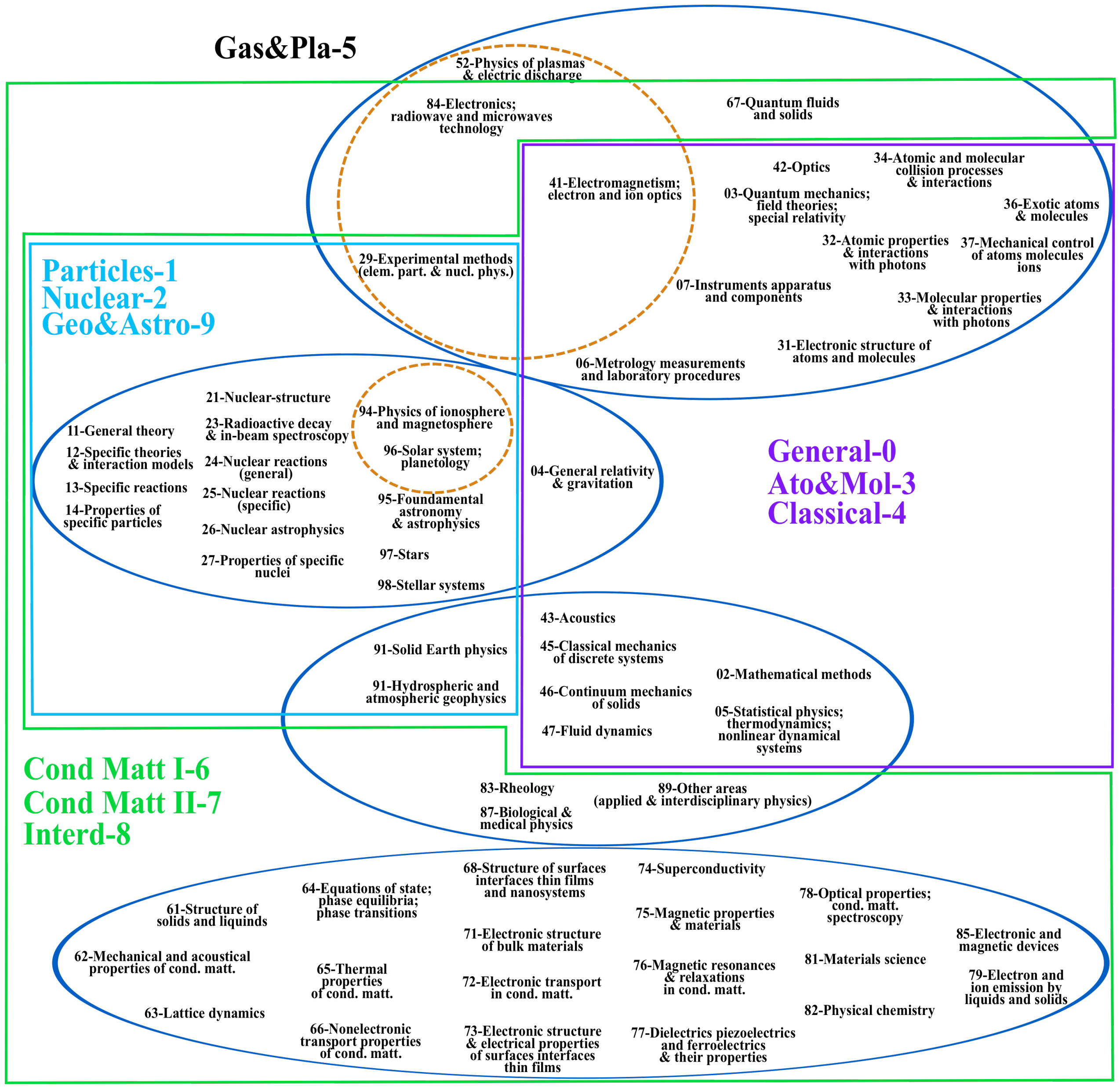
q_i^α → Community label of node i in layer α



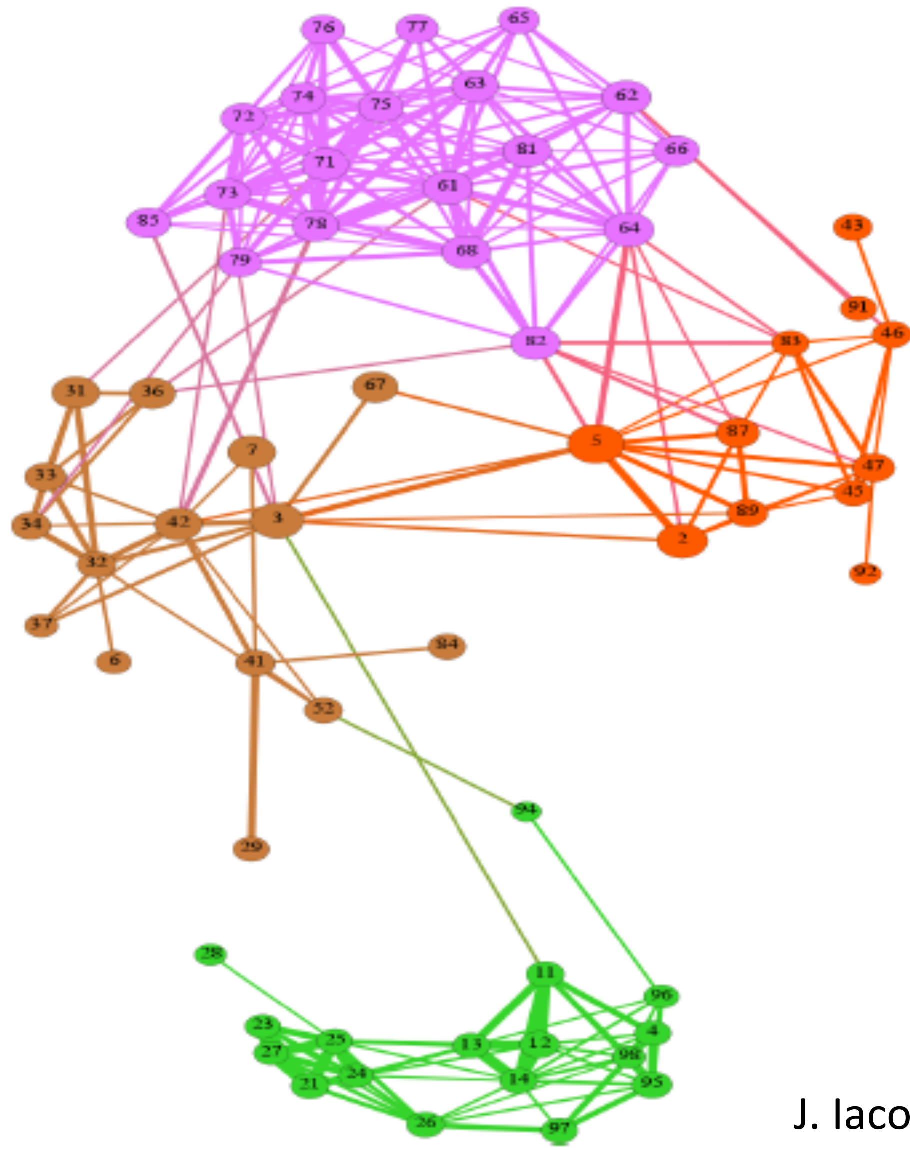


J. Iacovacci et al. Chaos 2016

Second PACS hierarchy (66 layers)



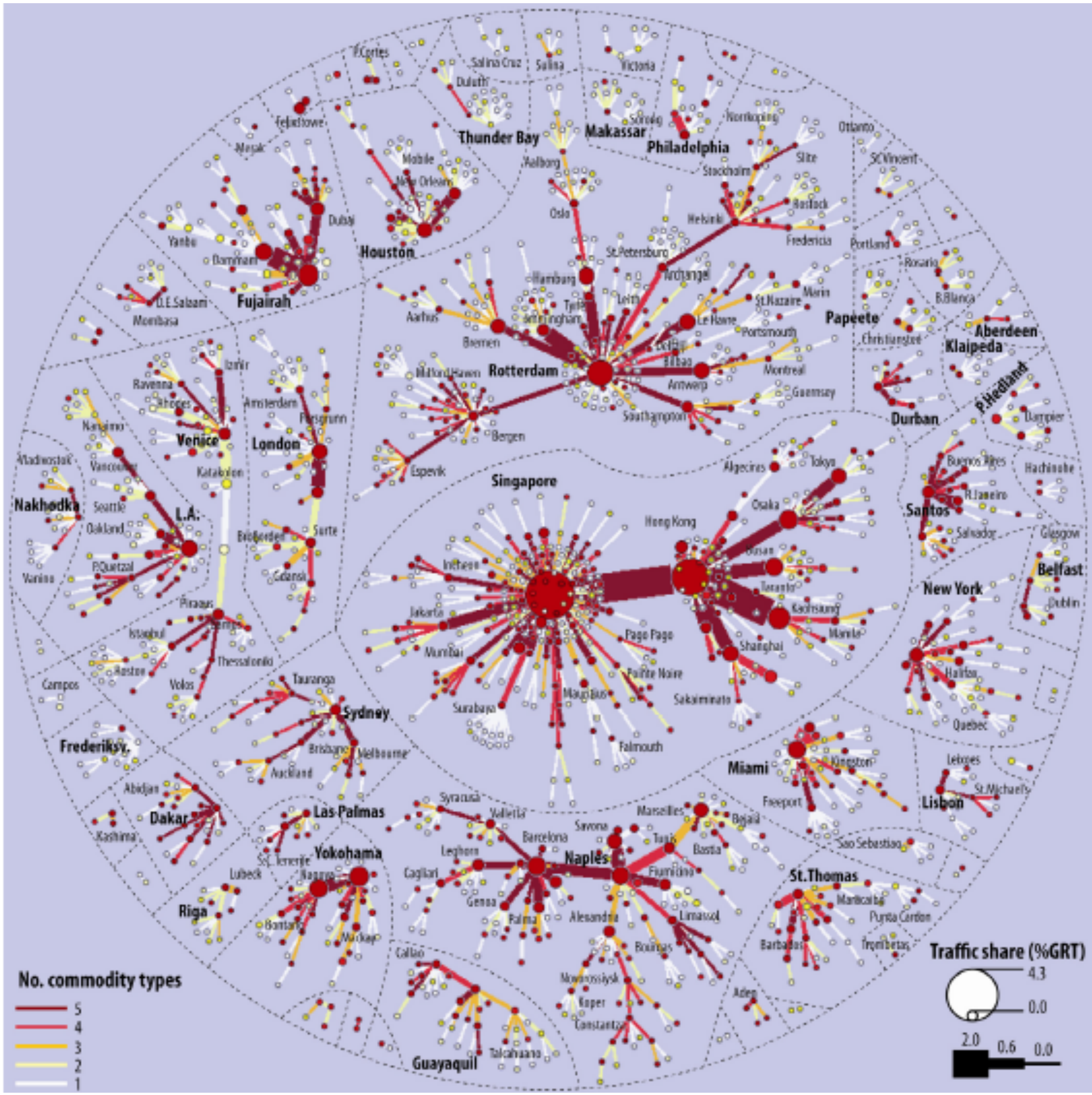
Network between the layers



J. Iacovacci et al. PRE 2015

Financial networks

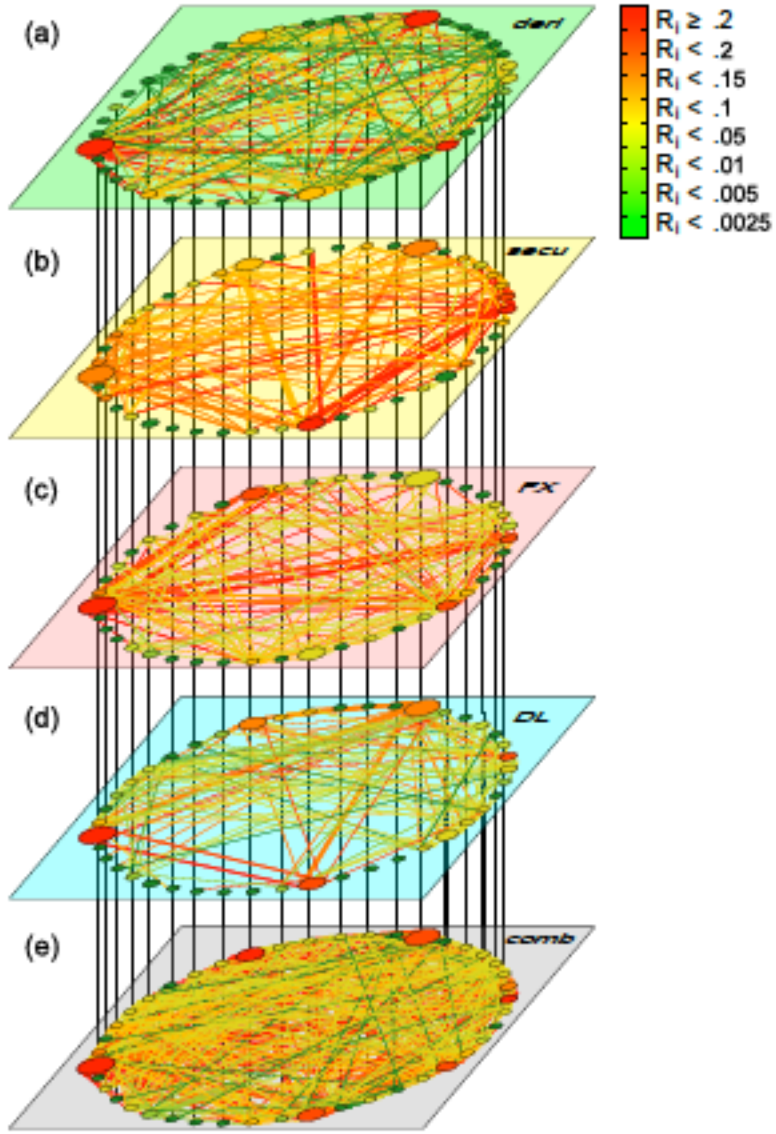
Trade networks



Systemic risk in multiplex financial networks

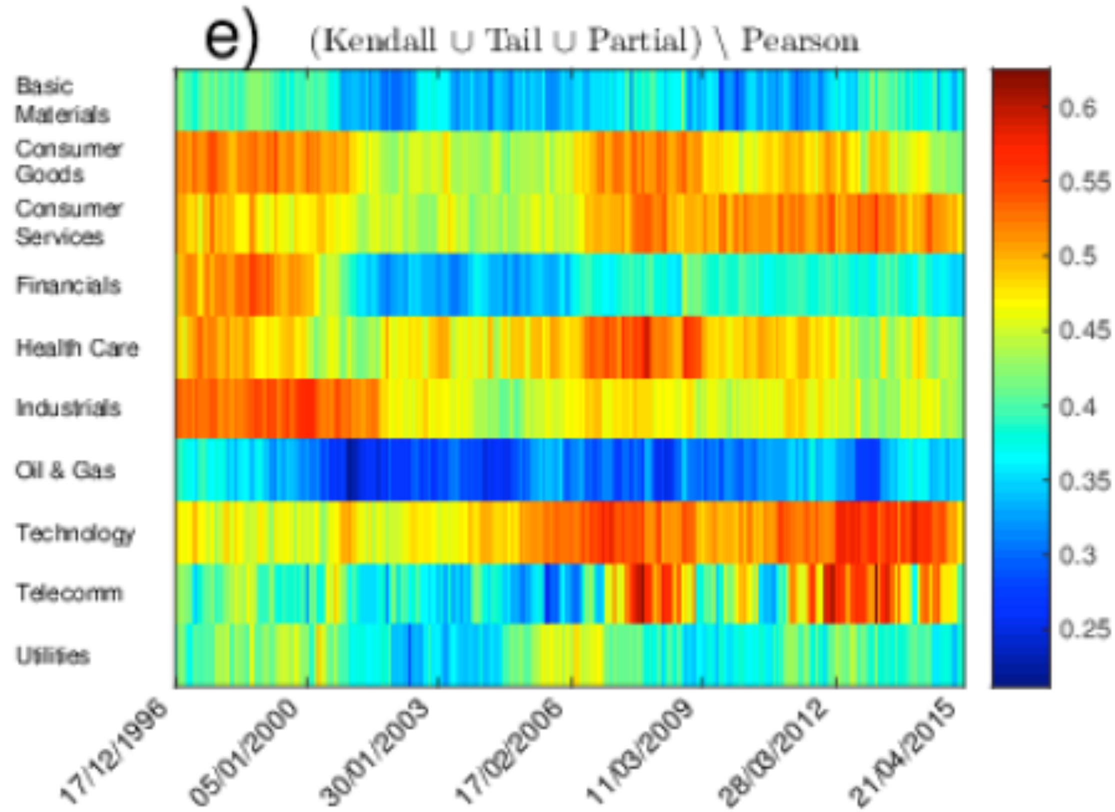
Banking multilayer in Mexico

- (a) exposure from derivatives
- (b) securities cross-holdings
- (c) foreign exchange exposure
- (d) deposits and loans
- (e) combined network.



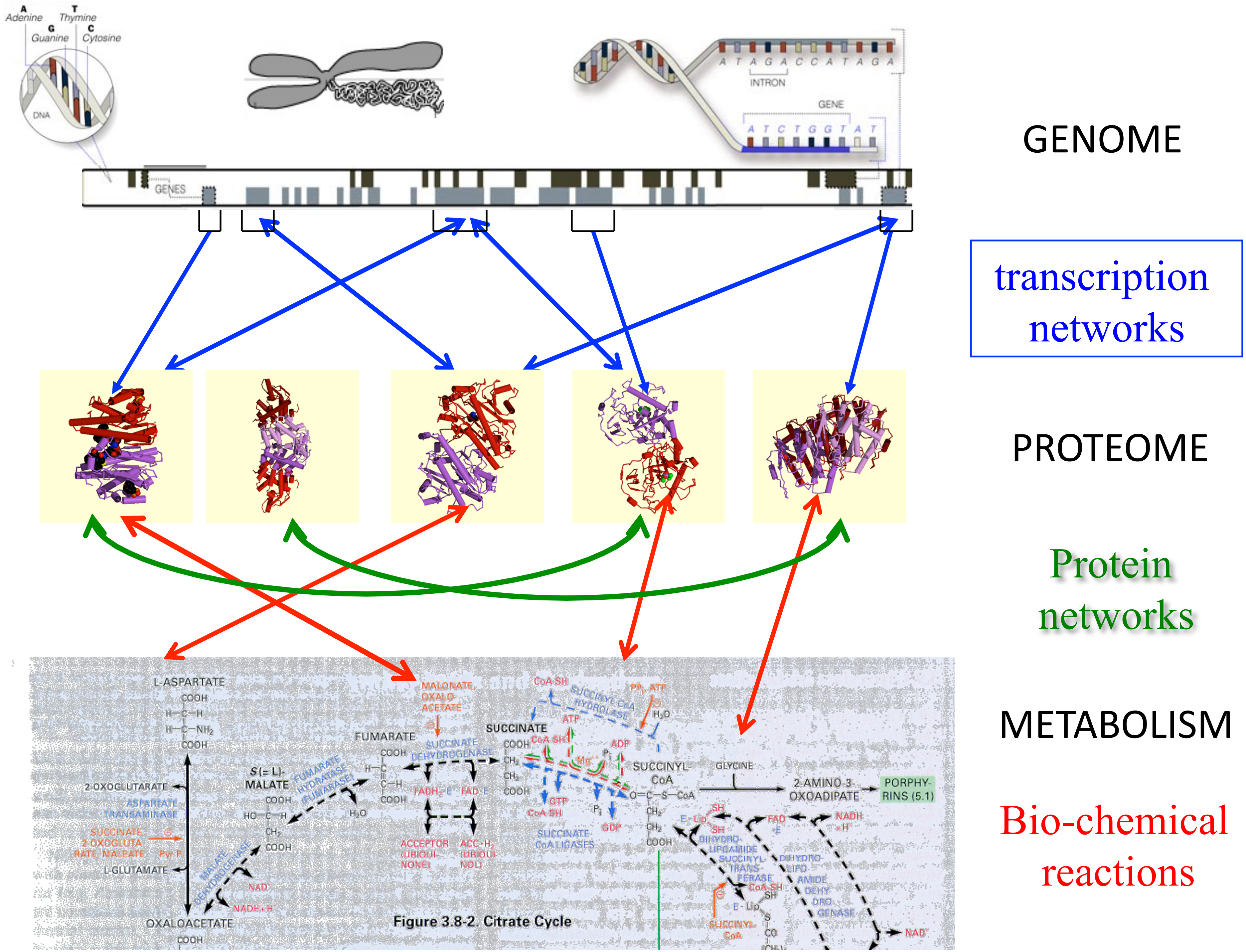
Poledna et al (2015)

Financial multiplex network using different measures of correlations between the assets



Musmeci et al. (2016)

Biological networks


GENOME
transcription networks
PROTEOME
Protein networks
METABOLISM
Bio-chemical reactions
Adenine
Thymine
Guanine
Cytosine
DNA
INTRON
GENE
GENES
Figure 3.8-2. Citrate Cycle

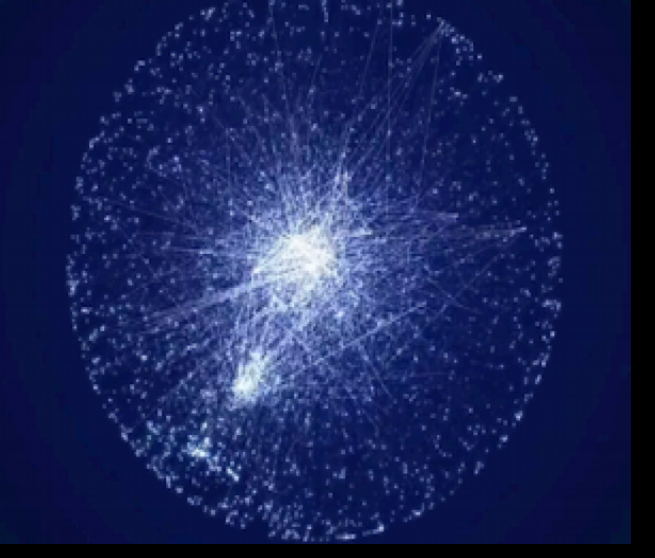
The present of network biology: The interactome

The interactome integrates

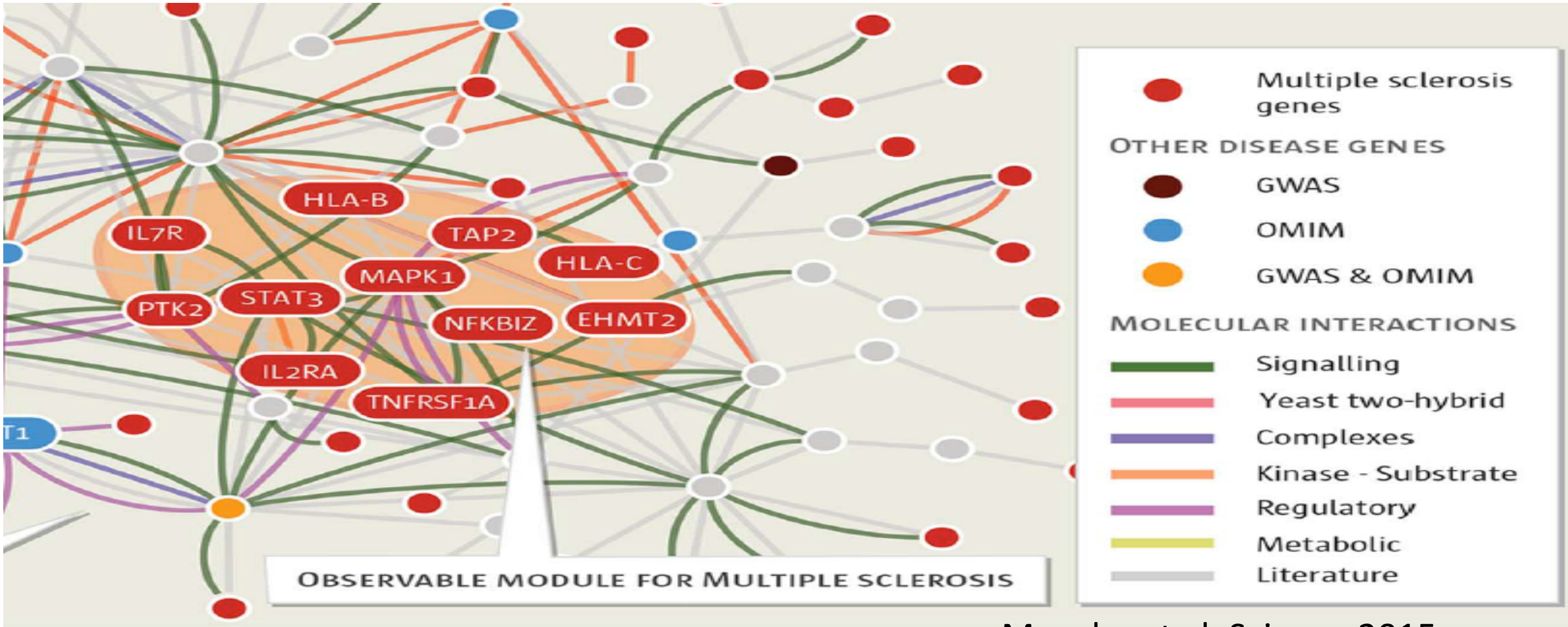
-Gene regulatory interactions
-Protein interactions
-Metabolic pathway interactions
-Kinase-substrate interaction
-Signaling interactions

with

-GWAS databases
-Online Mendelian Inheritance of Man



Menche et al. Science 2015



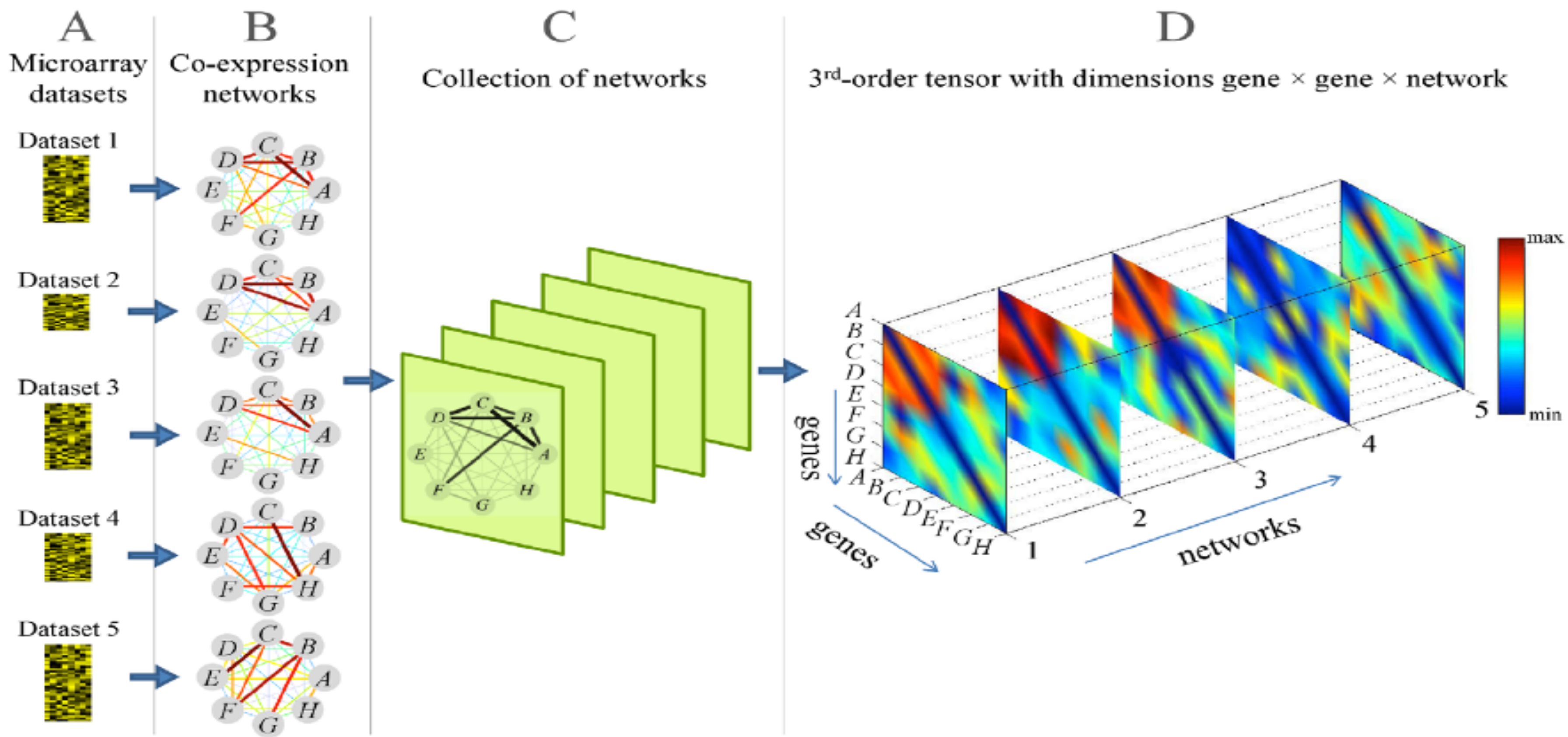
Menche et al. Science 2015

Network medicine

depends on the non-trivial interactions between

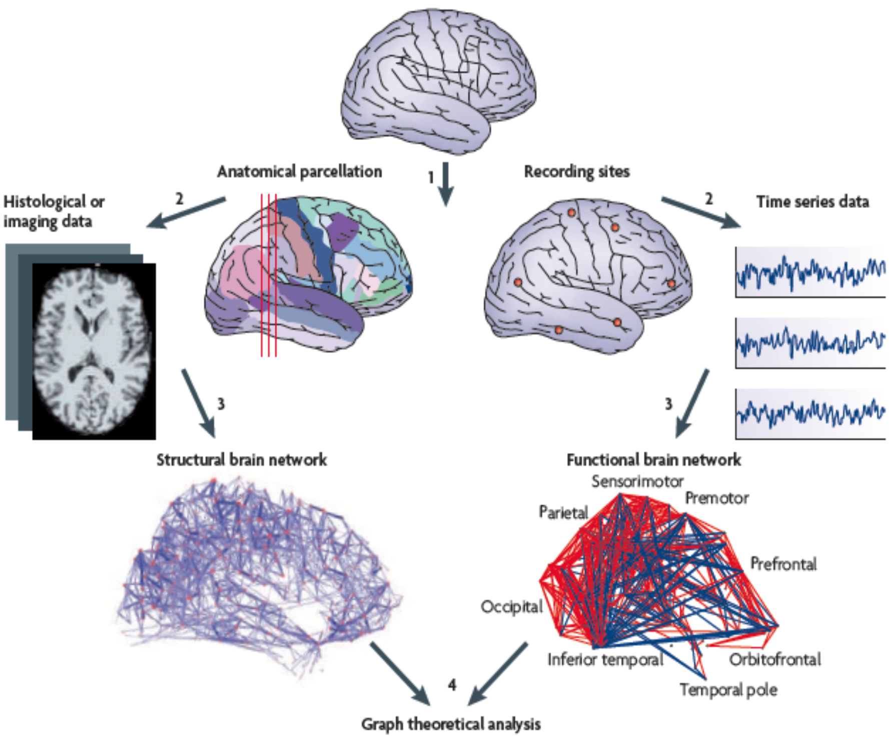
different networks

Multiplex networks
and gene expression datasets



Li et al. Plos Computational Biology 2011

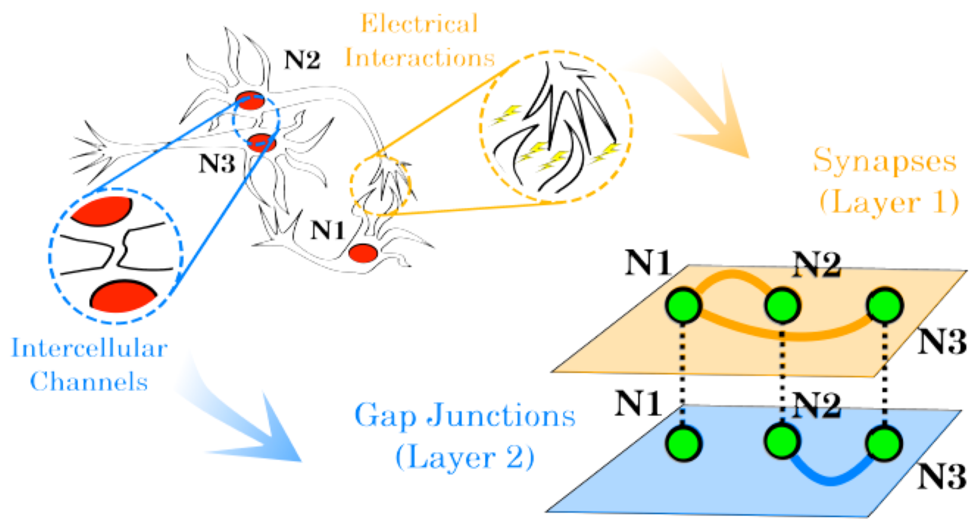
Interacting and multiplex Brain networks



The brain function is determined at the same time by the structural brain network and the functional brain network,

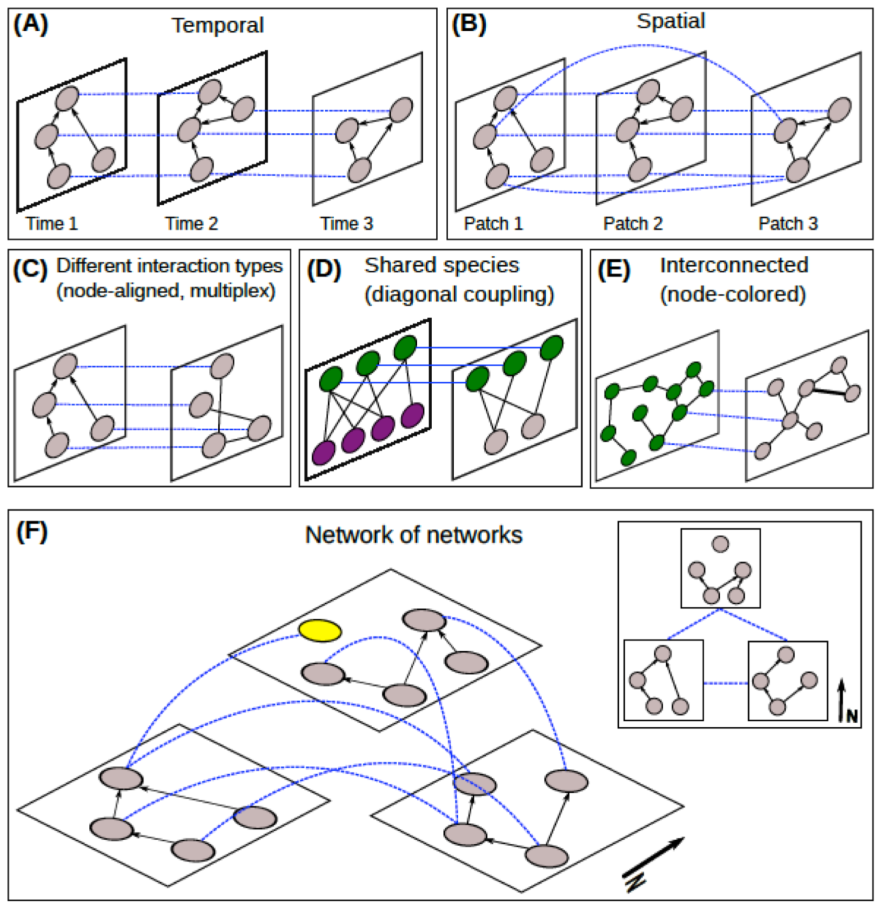
Bullmore Sporns 2009

Multiplex Connectome of C. elegans Gap junctions/Synapses



V. Nicosia and V. Latora PRE (2014)

Ecological networks



S Pilosof, MA Porter, S Kefi (2015)

Multilayers networks

In order to

characterize, model, predict and control

complex systems

we need to characterize

the structure

and the

the function

of

multilayer networks

New Review articles on Multilayer Networks

Journal of Complex Networks (2014) Page 1 of 69
doi:10.1093/comnet/cnu016

Multilayer networks

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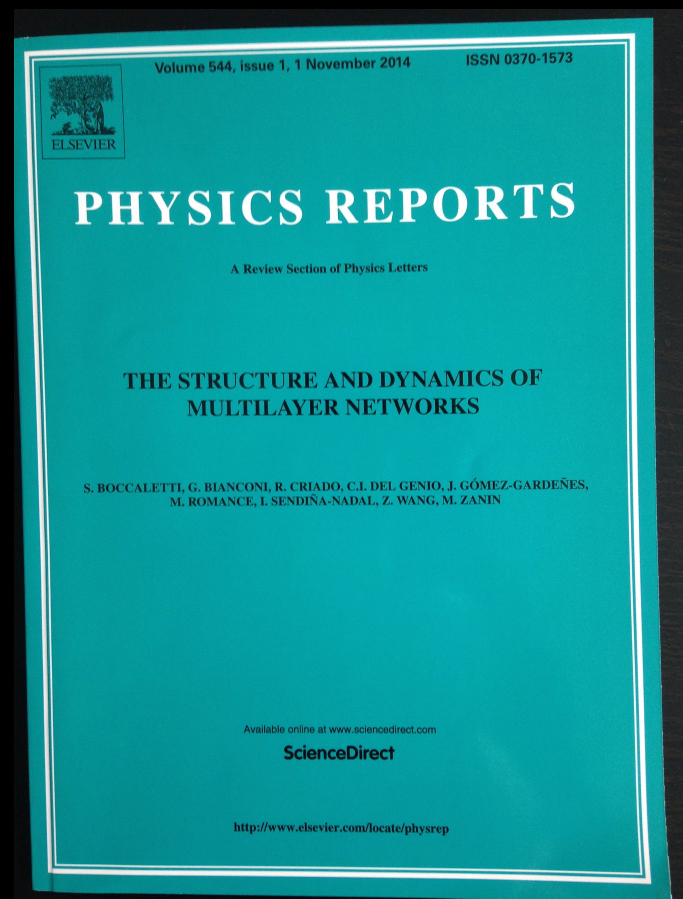
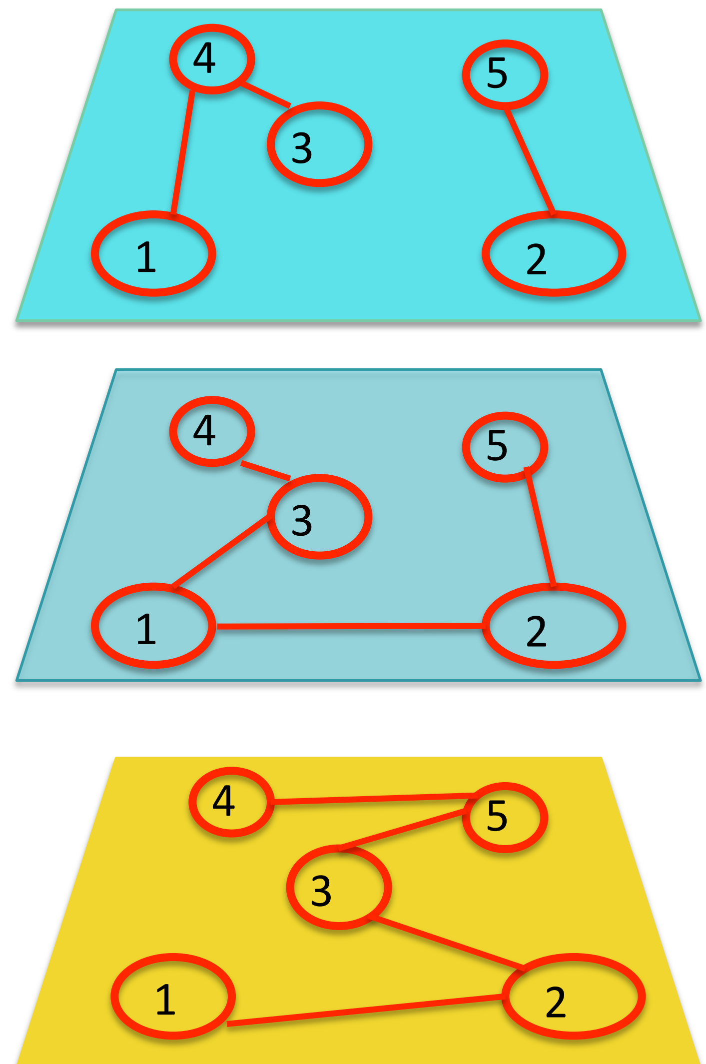
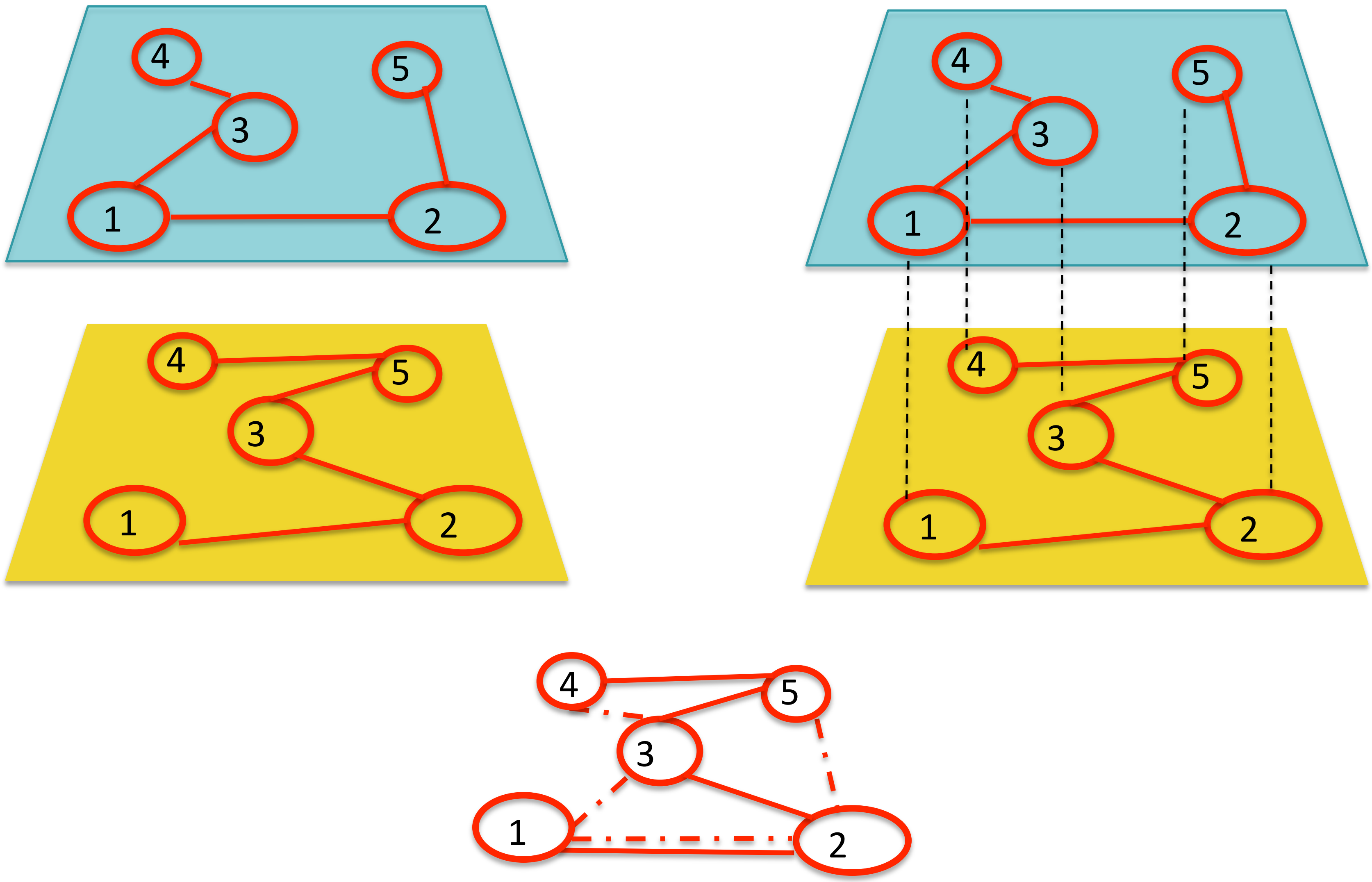


Multiplex Networks

- A multiplex is formed by a set of nodes that are present at the same time on different networks,
- A multiplex is formed by M layers (in the figure M=3)
- Each layer is formed by a network



Different representations of the same multiplex network



Representation of a multiplex

A multiplex network of N nodes formed by M layers

is fully specified by

M adjacency matrices

$$a^{[\alpha]}$$

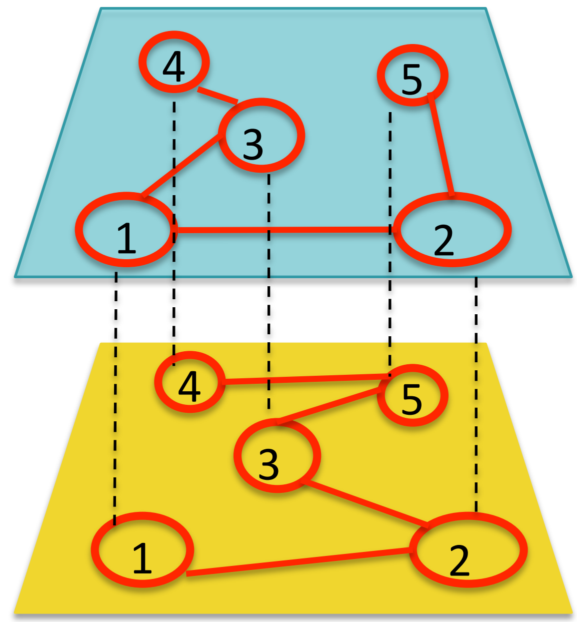
with α=1, 2, … M

of matrix elements

$$a_{ij}^{[\alpha]} = \begin{cases} 1 \text{ if node } i \text{ and node } j \text{ are linked in layer } \alpha \\ 0 \quad \text{otherwise} \end{cases}$$

Supra-adjacency matrix of a multiplex network

The supra-adjacency matrix includes all the links in each layers and the interlinks

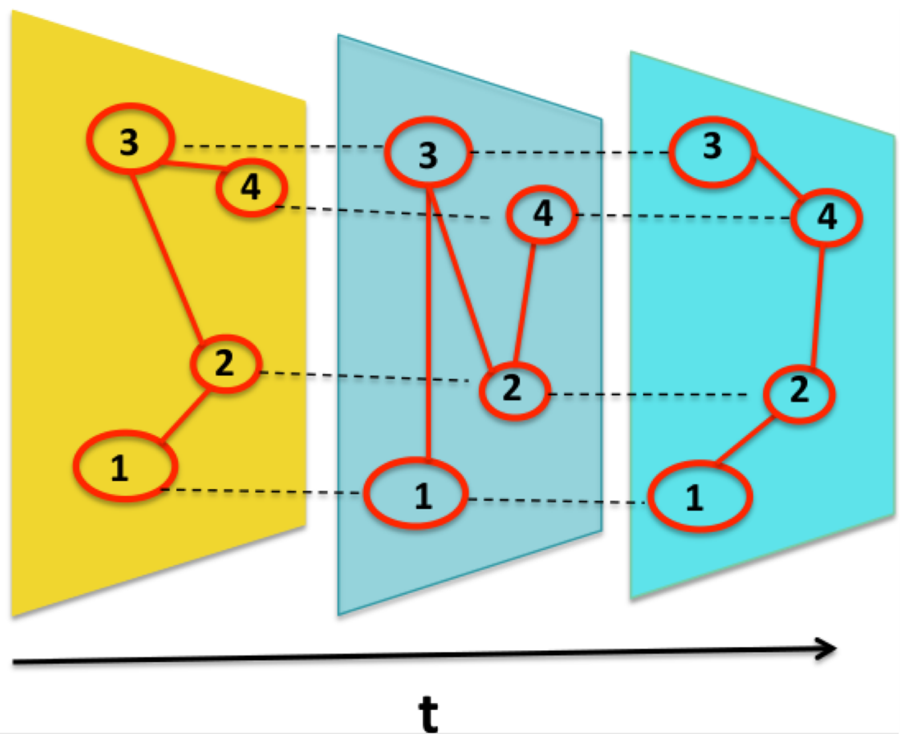


It indicates if a node i in layer α is connected to a node j in layer β

$$A_{i\alpha,\,j\beta} = \begin{cases} a_{ij}^{[\alpha]} & \text{if} \quad \alpha = \beta \\ \delta_{ij} & \text{if} \quad \alpha \neq \beta \end{cases}$$

$$\mathbf{A} = \left(\begin{array}{c|c|c|c} \mathbf{a}^{[1]} & \mathbf{I} & \cdots & \mathbf{I} \\ \hline \mathbf{I} & \mathbf{a}^{[2]} & \cdots & \mathbf{I} \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline \mathbf{I} & \mathbf{I} & \cdots & \mathbf{a}^{[M]} \end{array}\right),$$

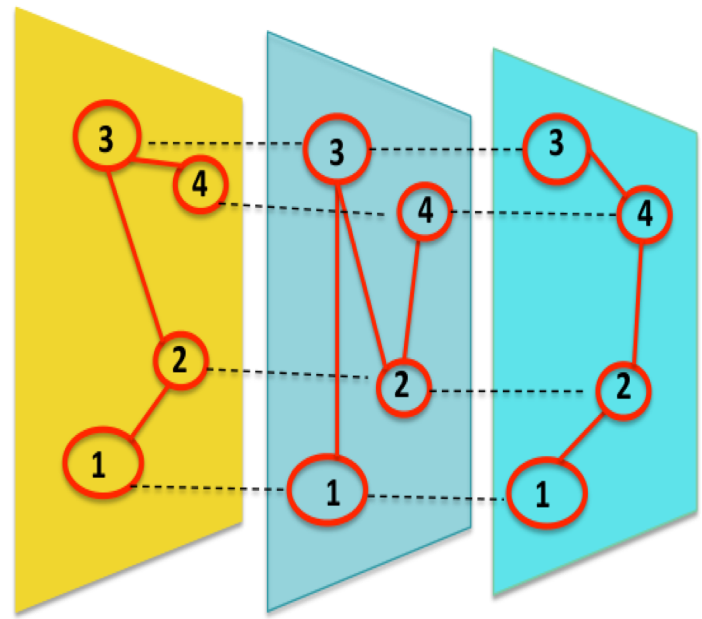
Temporal or multi-slice networks



Temporal networks can be seen as a multi-slice network where each slice is a temporal snapshot

Supra-adjacency matrix of temporal networks

(b)



The supra-adjacency matrix includes all the links in each layers and the interlinks

It indicates if a node i in layer α is connected to a node j in layer β

$$A_{i\alpha,j\beta} = \begin{cases} a_{ij}^{[\alpha]} & if \quad \alpha = \beta \\ \delta_{ij} & if \quad \alpha = \beta - 1 \end{cases}$$

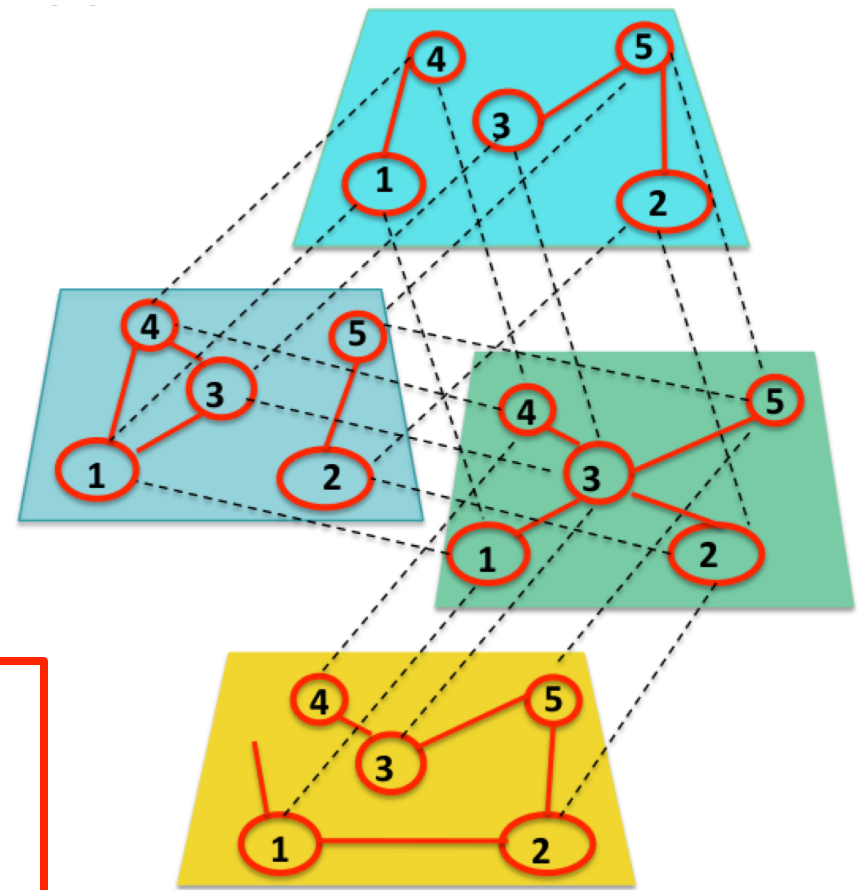
$$\mathbf{A} = \left(\begin{array}{c|c|c|c|c|c} \mathbf{a}^{[1]} & \mathbf{I} & 0 & \cdots & 0 & 0 \\ \hline 0 & \mathbf{a}^{[2]} & \mathbf{I} & \cdots & 0 & 0 \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline 0 & 0 & 0 & \cdots & \mathbf{a}^{[M-1]} & \mathbf{I} \\ \hline 0 & 0 & 0 & \cdots & 0 & \mathbf{a}^{[M]} \end{array}\right),$$

Network of networks

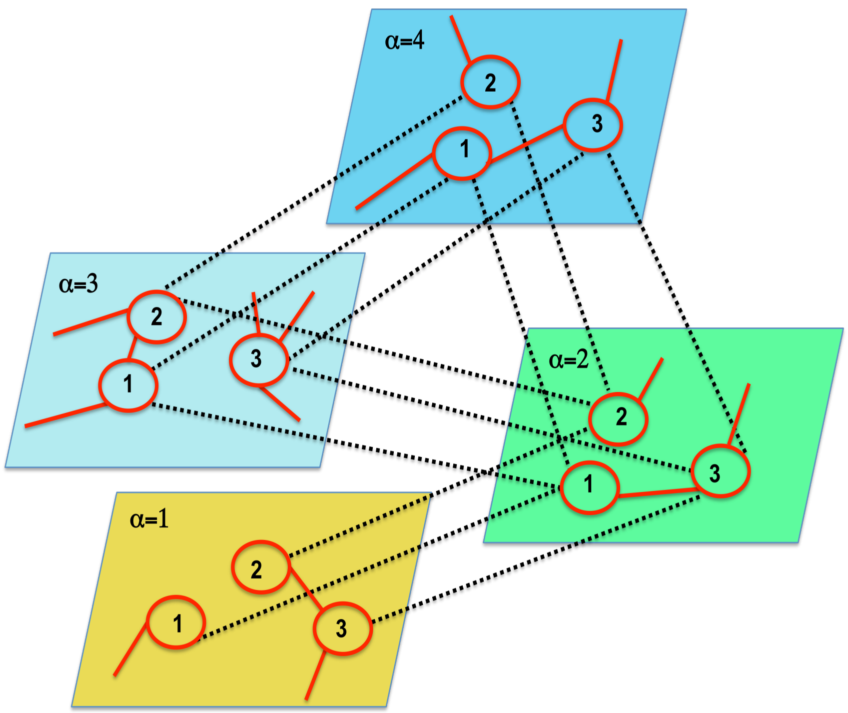
Each node is determined by the pair of label (i,α)
$i=1,2,..,N$ $\alpha=1,2,..,M$

Links across layers are called interlinks

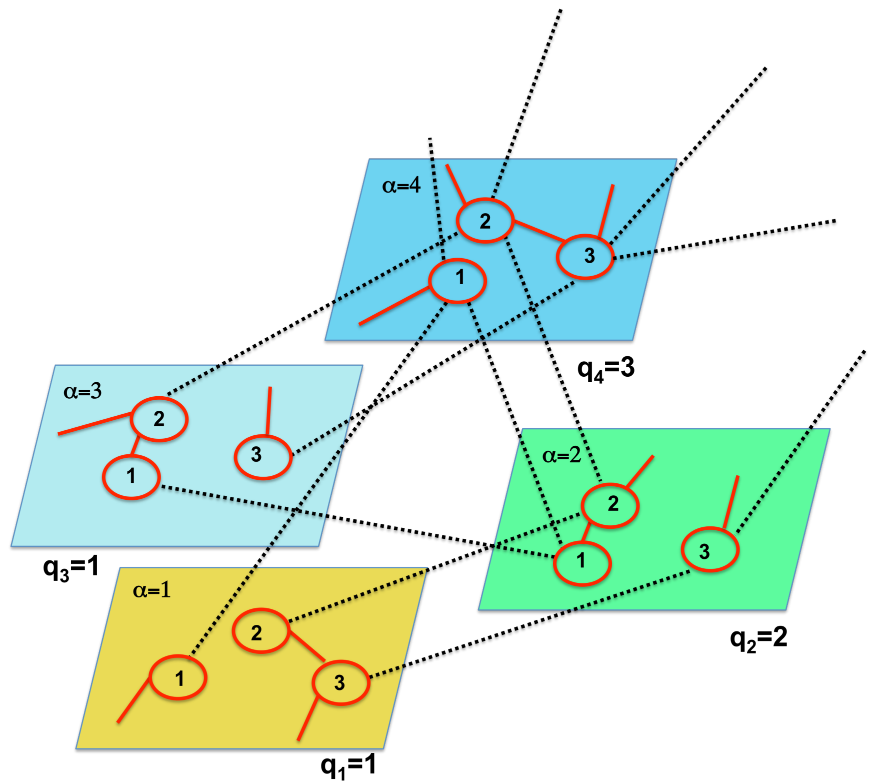
$$A_{i\alpha,\, j\beta} = \begin{cases} 1 & \text{if } (i,\alpha) \text{ and } (j,\beta) \text{ are linked} \\ 0 & \text{otherwise} \end{cases}$$



Types of network of networks

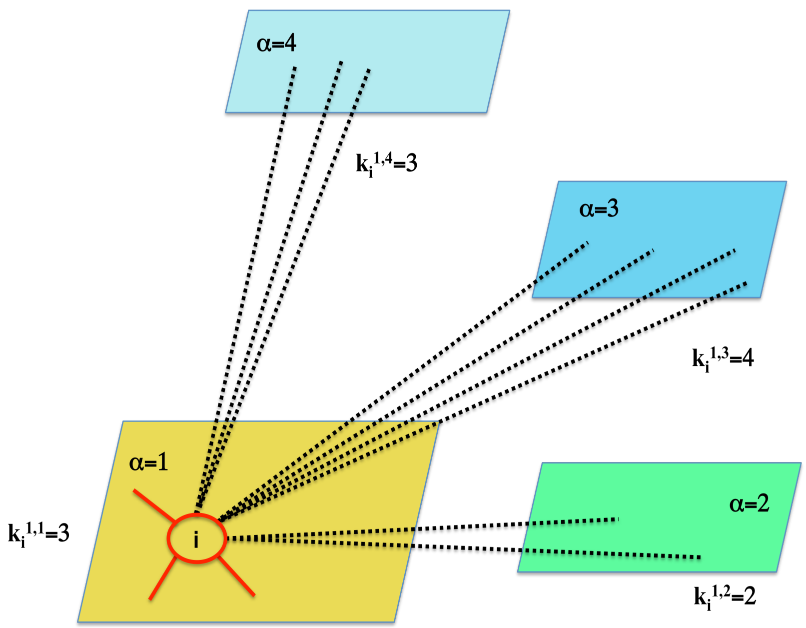


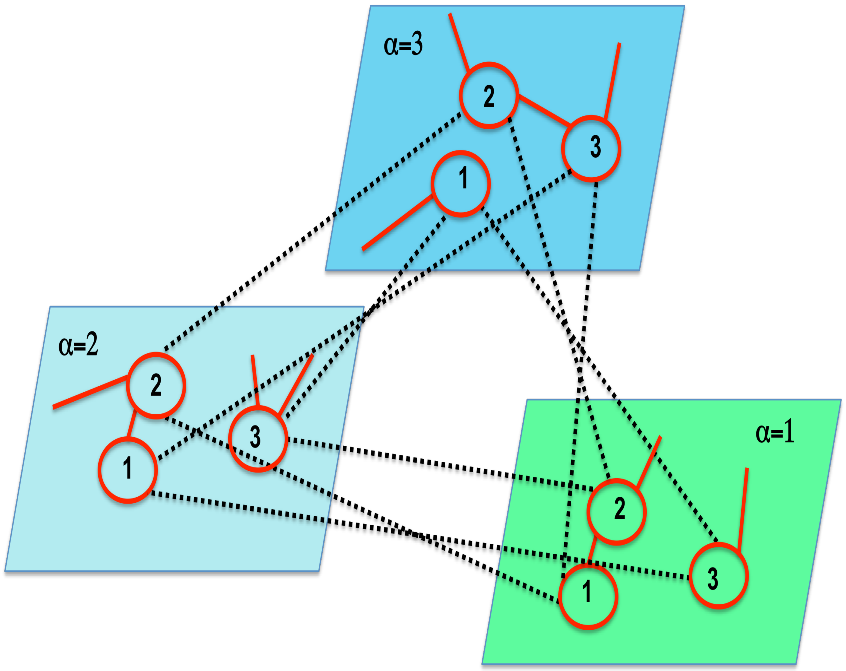
with replica nodes



configuration model

Types of network of networks





with multiple interlinks

random matching of the nodes

Conclusions

Most complex systems
are formed by
several interacting and co-evolving networks.
In order to
model, predict and control
complex networks
we need to investigate the interplay between
structure and dynamics in
Multilayer Networks

References and repositories

- **REVIEWS**

- Boccaletti, Stefano, Ginestra Bianconi, et al. "The structure and dynamics of multilayer networks." *Physics Reports* 544, no. 1 (2014): 1-122.

- Kivelä, Mikko, Alex Arenas,et al. "Multilayer networks." *Journal of complex networks* 2, no. 3 (2014): 203-271.

- **DATA and CODES repositories :**

- GitHub page: https://github.com/ginestrab (G. Bianconi)
- GitHub page: https://github.com/manlius (M. De Domenico)
- GitHub page: https://github.com/KatolaZ (V. Nicosia)

- http://deim.urv.cat/~manlio.dedomenico/data.php (M. De Domenico)