Mathematics of Crime

an intensive course by Andrea Bertozzi
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There is an extensive applied mathematics literature developed for problems in the biological and physical sciences. Our understanding of social science problems, from a mathematical standpoint, is less developed but also presents some very interesting problems especially for young researchers. This set of lectures uses urban crime as a case study for interesting social science applications and covers a variety of mathematical methods that are applicable to such problems. The course will blend some basic tutorials for graduate students with some recent research results on mathematics of crime. From a modeling standpoint we will cover methods in linear and nonlinear partial differential equations, variational methods for inverse problems, models from statistical mechanics and methods from statistics such as point process models. From an application standpoint we will consider the development of spatiotemporal event patterns for different classes of crimes.

Rather than address many different sociological issues, we focus on crime patterns that can be studied by routine activity theory – namely that offenders are led to choose targets based routine activities they perform throughout their day, leading to natural classes of models that build on prior work in ecology, population biology, and statistical physics. At the same time, it is very important to understand the role of modern statistical methods in analyzing real police data. The course will consider both ``bottom up'' and ``top down'' approaches to understanding crime patterns and how the two approaches could lead to converge to a unifying theory. The course will be structured around the following subtopics:

Agent-based models: we consider basic lattice models from statistical physics and discuss how one might `course grain' such models using nonlinear partial differential equations – we discuss both theory and numerical results for when mean field approximations are appropriate for agent based models. We will discuss some recent work using agent-based models to study repeat offenders of residential burglaries.

Mean field limit: we consider continuum limits of mean field models including a tutorial of basic mathematical methods for such mean field models. We review some simpler examples such as the heat equation resulting from a random walk, and how such models generalize to more complex agent-based models. We discuss techniques for analyzing nonlinear mean field models including linear stability theory and weakly nonlinear bifurcation theory. We discuss some basic issues associated with numerical solution of mean field models. Current research on formation of crime hotspots will be discussed along with the mathematical theory.

Mathematics of real time decisions: An important issue that arises in predictive policing is the ability to make real time decisions based on streaming data. If a crime upswing is taking place in a neighborhood, when is a decision made to send more patrols to that neighborhood? Such data is often only partially complete and is subject to a lot of random noise. At the same time, this is a basic question that comes up in many different application areas ranging from DNS internet attacks to weather forecasting. A classical statistics problem is to develop filtering methods that minimize average detection delay while simultaneously minimizing the false alarm rate. We discuss some of the basic mathematics associated with statistical filtering of data such as Kalman filter and change point detection.

Statistical density estimation: We present a lecture on solution of spatial inverse problems using variational methods. Statistical density estimation is a good case study that arises in the study of geographic placement of criminal events. Using data from Los Angeles we contrast more classical methods such as approximation by Gaussian kernels with more recent methods such as total variation regularization and data fusion methods that incorporate additional data such as census data. We compare and contrast the density estimation problem with some related problems that arise in image processing including denoising and image inpainting.

Geographic profiling: Geographic profiling involves estimating the location or home base of an unknown offender by locations of crimes known to be tied to the suspect. Mathematical issues associated with geographic profiling include Bayesian statistics, Fokker-Planck equations for probability density functions, and fractional diffusion.

Point process models: We review some of the basic mathematics of point process models starting with the Poisson arrival process and how to measure its statistics in real world data. We contrast with self-exciting point process models such as the Hawkes process and discuss the role of memory in such problems.

Gang rivalries: We consider the problem of dynamics of rivalries of street gangs. One such concrete example involves the Hollenbeck division of the Los Angeles Police Department home to almost 30 gangs. This topic introduces some additional mathematical tools such as graph theory and network structure. We discuss how these ideas combine with some of the above methods such as point process models for gang retaliatory activity and the role of gang turf in formation of rivalries and dynamics of criminal events.

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(partial list from UCLA researchers)

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