# Measure Theory: Exercises 1

1. Consider the collection $\mathcal{A}$ of subsets $A_1, A_2, \ldots$ of the integers such that $A_i = \{ni \mid n \text{ is an integer}\}$.
Determine what is $\sigma(\mathcal{A})$.

2. Give an example of a decreasing sequence $B_1 \supset B_2 \supset \cdots$ such that none of the $B_i$ has finite measure and $\lim_{i\to\infty} \mu(B_i) \neq \mu(\cap_{i=1}^{\infty} B_i)$.

3. If $A_1, \ldots, A_n$ are measurable sets each of finite measure show that $\mu(\cup_{i=1}^{n} A_i) = \sum_{S\subseteq\{1,2,\ldots,n\}} (-1)^{|S|+1} \mu(\cap_{i\in S} A_i)$.

4. Determine the smallest sigma algebra on $\mathbf{R}$ that is generated by the collection of all one-point subsets of $\mathbf{R}$.