ltcc1.tex Week 2 7 October 2013

PROBLEMS 2

Q1 (*Generalised Pythagoras theorem*). A right-angled triangle has sides 1 (the hypotenuse), 2 and 3. A semicircle (or any other plane shape) of area A_1 is drawn with base side 1; similar copies of this are drawn with bases sides 2 and 3, with areas A_2, A_3. Show that

$$A_1 = A_2 + A_3.$$

Deduce Pythagoras' theorem on taking these shapes to be squares.

Q2 (*Rejection method*). (i) The *subgraph* of a probability density function f is $\{(x, y) : y \leq f(x)\}$. Show that X has density f iff X is the first coordinate of a point (X, Y) uniformly distributed over the subgraph of f.
(ii) Suppose that we wish to sample from a density f, and that $f \leq cg$ for some $c > 0$ and density g that we know how to sample from. Show that the algorithm
(a) simulate X from g;
(b) given $X = x$, simulate $Y = Ug(x)$, where U has the uniform distribution $U(0, 1)$ and is independent of X;
(c) *reject* the point (X, Y) if $Y > f(x)$
(d) record the x-coordinates of accepted points
gives a sample with density f.

NHB