PROBLEMS 2

Q1 (Generalised Pythagoras theorem). A right-angled triangle has sides 1 (the hypotenuse), 2 and 3. A semicircle (or any other plane shape) of area $A_1$ is drawn with base side 1; similar copies of this are drawn with bases sides 2 and 3, with areas $A_2$, $A_3$. Show that

$$A_1 = A_2 + A_3.$$ 

Deduce Pythagoras’ theorem on taking these shapes to be squares.

Q2 (Rejection method). (i) The subgraph of a probability density function $f$ is $\{(x, y) : y \leq f(x)\}$. Show that $X$ has density $f$ iff $X$ is the first coordinate of a point $(X, Y)$ uniformly distributed over the subgraph of $f$.

(ii) Suppose that we wish to sample from a density $f$, and that $f \leq cg$ for some $c > 0$ and density $g$ that we know how to sample from. Show that the algorithm

(a) simulate $X$ from $g$;
(b) given $X = x$, simulate $Y = Ug(x)$, where $U$ has the uniform distribution $U(0, 1)$ and is independent of $X$;
(c) reject the point $(X, Y)$ if $Y > f(x)$
(d) record the $x$-coordinates of accepted points

gives a sample with density $f$.

NHB