Methods of Industrial Mathematics – Problem Sheet on Session 4.

Issue No: 1.0

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Date: 08/03/2018

1. Find the stationary points of

$$f(x) = x^2(1 - x^2),$$

and determine if they are maxima or minima and what the maximum and minimum values are.

2. (i) Prove that

$$f(x, y) = (x - 2)^2 + (y - 3)^2$$

is a minimum at $x = 2, y = 3$ (as is obvious by inspection!) by finding the stationary point and applying the criteria for second derivatives.

(ii) Now in two separate ways including one using a Lagrange multiplier show that minimising $f(x, y)$ subject to the constraint $y = 2x$ yields a stationary (minimum) value of $\frac{1}{5}$.

3. Minimise $f(x) = (x - 7)^2$ subject to $0 \leq x \leq 4$.

4. Prove that $f(x) = x^4 + x^2 + 1$ takes a minimum value of 1 at $x = 0$. Now form

$$g(x, y;\ \lambda) = y^2 + y + 1 + \lambda(y - x^2);$$

and show that with respect to variation of x, y it takes the stationary value

$$d_\lambda = 1 - \frac{1}{4}(\lambda + 1)^2.$$

It is apparent that this is a lower bound on the minimum of $f(x)$, which takes a maximum value of 1. This is a simple example of a complementary extremum principle.

5. **Challenge Question**. Perform the orthogonality of axes analysis done in the slides for two dimensions this time in three dimensions.