Proposed Syllabus for an LTCC Intensive Course on ***Turbulent Flows***

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6th May 2013

Turbulence remains as *the* unsolved problem in classical (fluid) mechanics. Standard approaches comprise dimensional analysis, semi-empirical modelling of the unclosed terms in the Reynolds-averaged Navier–Stokes equations (RANS), and methods aiming at the resolution of the abundance of scales involved, as Large-Eddy or Direct Numerical Simulation (LES, DNS). Notwithstanding their undeniable ongoing progress, many aspects of the important class of wall-bounded and separated shear flows at large Reynolds numbers, Re, yet lack a sound theoretical understanding. However, advanced asymptotic techniques have proven promising more recently. This course focuses on their impact on classical statistical/modelling methods and also novel findings:

- **Summary of governing equations:** Navier–Stokes, energy, and transport equations for vorticity and related quantities. On vortex dynamics (Helmholtz and Biot–Savart theorems). Validity of Newtonian constitutive law and no-slip condition for high-Re flows.
- **Statistical description of turbulent flows:** Reynolds decomposition. Equivalence of Reynolds (ensemble) and time averaging for stationary processes (*ergod theorem*). Central moments, one-/multi-point, auto-/cross-correlations. Spectra. Energy cascade, associated hierarchy of spatial and temporal scales. Local isotropy. Kolmogorov theory: similarity hypotheses, matching inertial/viscous ranges (inertial subrange, $5/3$-power law).
- **Principles of averaging, modelling, simulation:** Classical Reynolds- and Favre-averaging. Systematic derivation of transport equations for one-point correlations. Resulting hierarchy of *incomplete* zero-/one-half-equation (algebraic), one- and one-and-a-half-equation and *complete* two-equation (k-ϵ, k-ω) and full Reynolds stress closures – advantages and shortcomings. Numerical resolution of scales, status quo: RANS vs. LES vs. DNS.
- **Asymptotic theory of turbulent shear flows:** Separation of scales and their above hierarchy reappraised. Equivalence of Kolmogorov microscales and viscous-sublayer scales. Approach to turbulent dynamics (*multiple scaling*) and averaged flow (*matched asymptotic expansions*). *Internal* (fully developed pipe/channel) and *external* (boundary layer and free shear) flows. Classical high-Re asymptotic theory of turbulent boundary layers and its two-parameter extensions – resolving a seeming dilemma in perturbation analysis.

Some suggested reading

Tennekes, H., Lumley, J. L.: *A First Course in Turbulence.* MIT Press, Cambridge (MA). 1972.
Schlichting, H., Gersten, K.: *Boundary-Layer Theory* (8th ed.), Parts III–V. Springer. 2003.