Wavelets/Multiscale Methods in Statistics

LTCC Intensive course

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Wavelets are mathematical functions which, when plotted, resemble "little waves": that is, they are compactly or almost-compactly supported, and they integrate to zero. This is in conrast to "big waves" – sines and cosines in Fourier analysis, which also oscillate, but the amplitude of their oscillation never changes.

Wavelets are useful for decomposing data into "wavelet coefficients", which can then be processed in a way which depends on the aim of the analysis. One possibly advantageous feature of this decomposition is that in some set-ups, the decomposition will be *sparse*, i.e. most of the coefficients will be close to zero, with only a few coefficients carrying most of the information about the data. One can imagine obvious uses of this fact, e.g. in image compression.

The decomposition is particularly informative, fast and easy to invert if it is performed using wavelets at a variety of *scales* and *locations*: hence the term "multiscale". The role of scale is similar to the role of frequency in Fourier analysis. However, the concept of location is unique to wavelets: as mentioned above, they are localised around a particular point of the domain, unlike Fourier functions.

The course will provide a self-contained introduction to the applications of wavelets in statistics and attempt to justify the extreme popularity which they have enjoyed in the literature over the past 15 years. The syllabus will be as follows.

1. Motivation: what are "multiscale methods" and how can they be useful in statistics? What are wavelets and how do they achieve a multiscale view of the data? Examples of nonparametric data smoothing via wavelets.

2. Introduction to wavelets: mathematical construction and properties, continuous wavelet transform, examples of wavelets, multiresolution analysis, discrete wavelet transform, applications of wavelets in the real world.

3. Wavelets for nonparametric function estimation: thresholding, why does it work, proof of mean-square consistency, noise-free reconstruction property.

4. Special topics: Unbalanced Haar wavelets, Haar-Fisz transforms for variance stabilisation, market volatility estimation via Haar-Fisz.