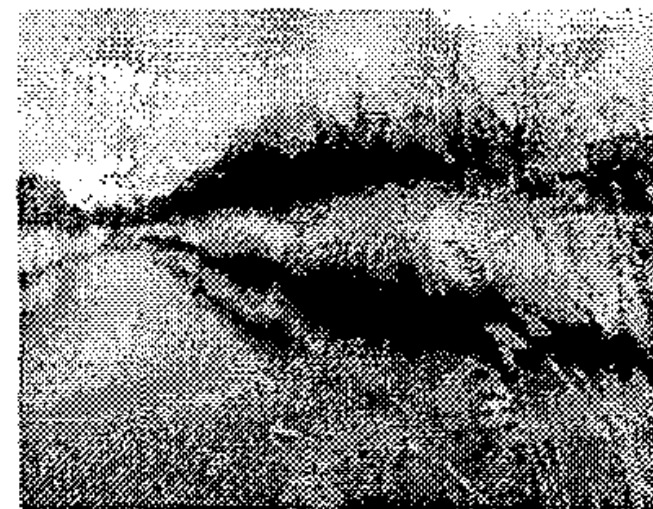
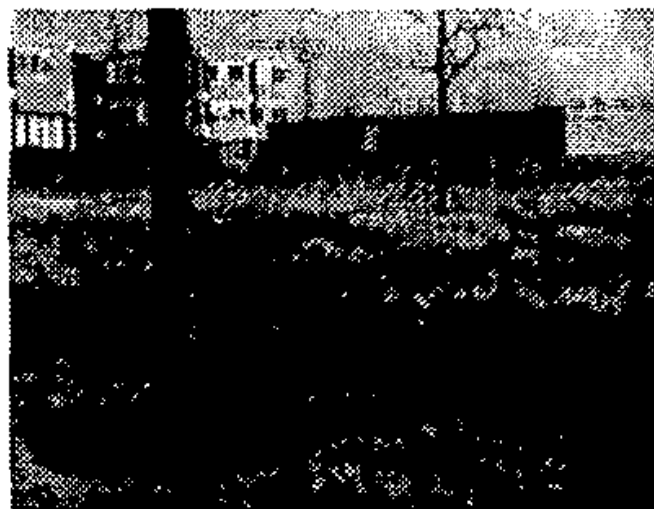
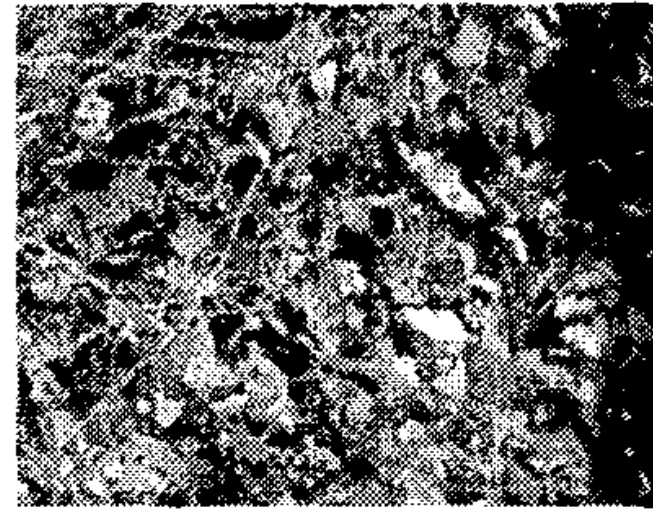


Two Large Scale Structures

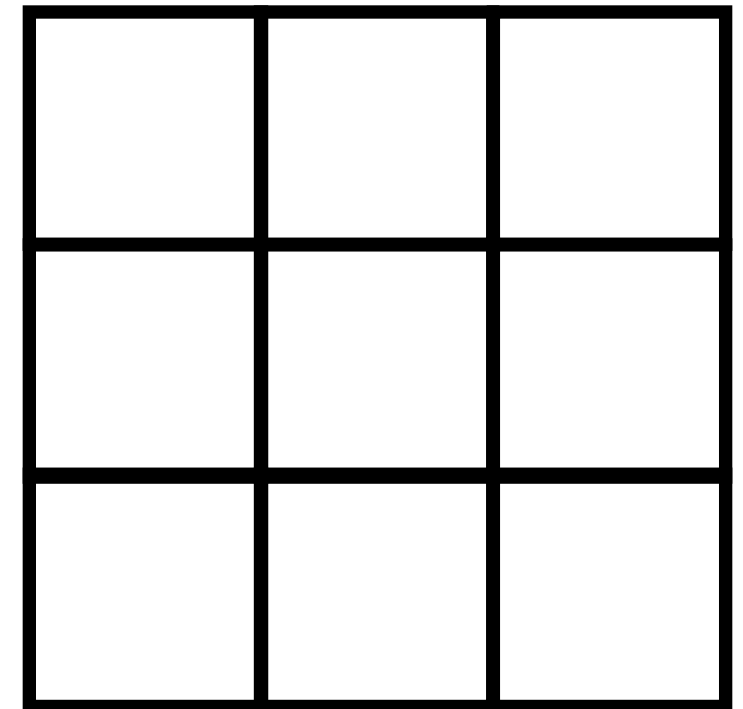
Space of Natural Images

- Mumford Data Set (1999)



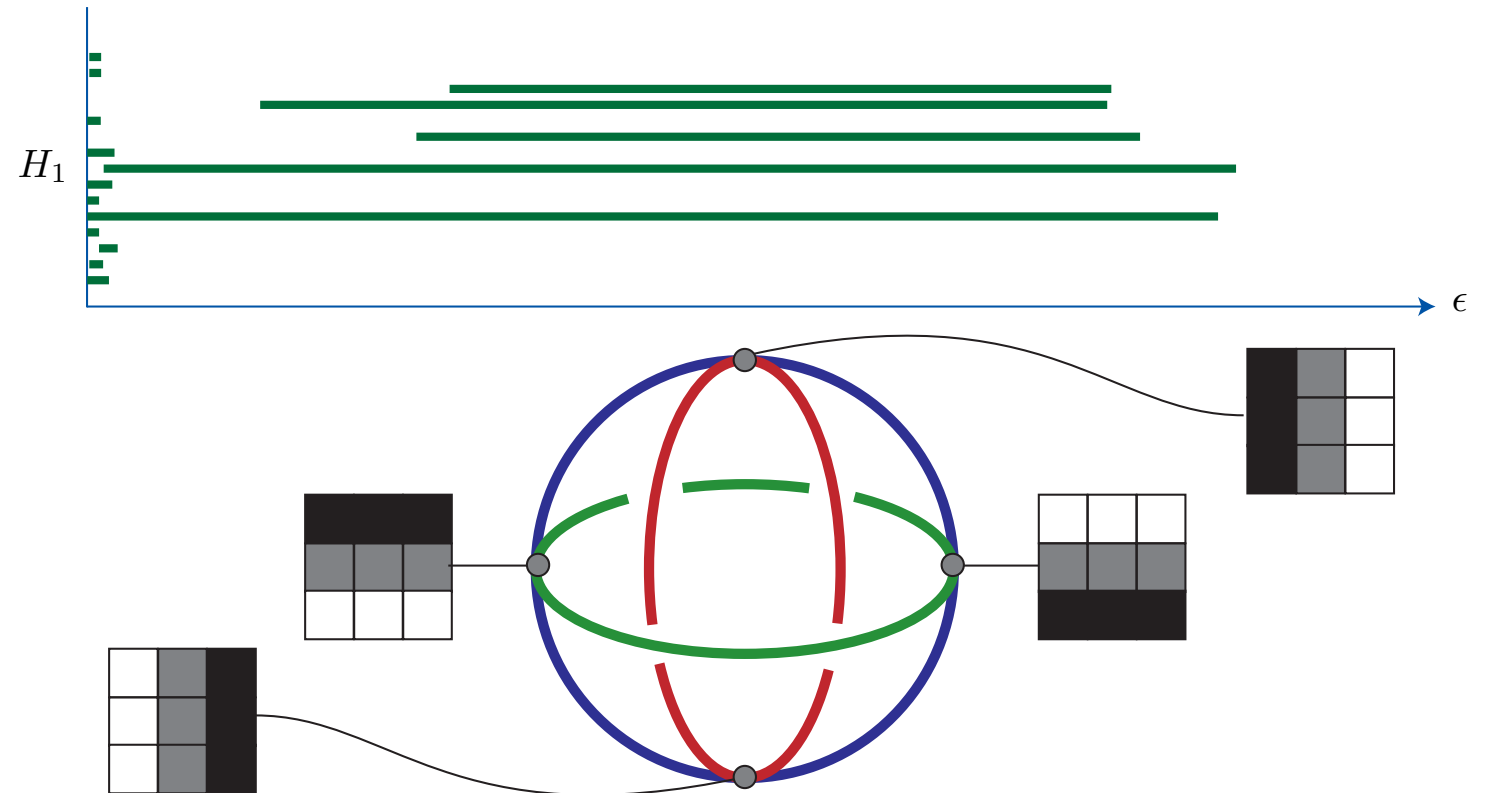
Space of Natural Images

- 3x3 patches



Space of Natural Images

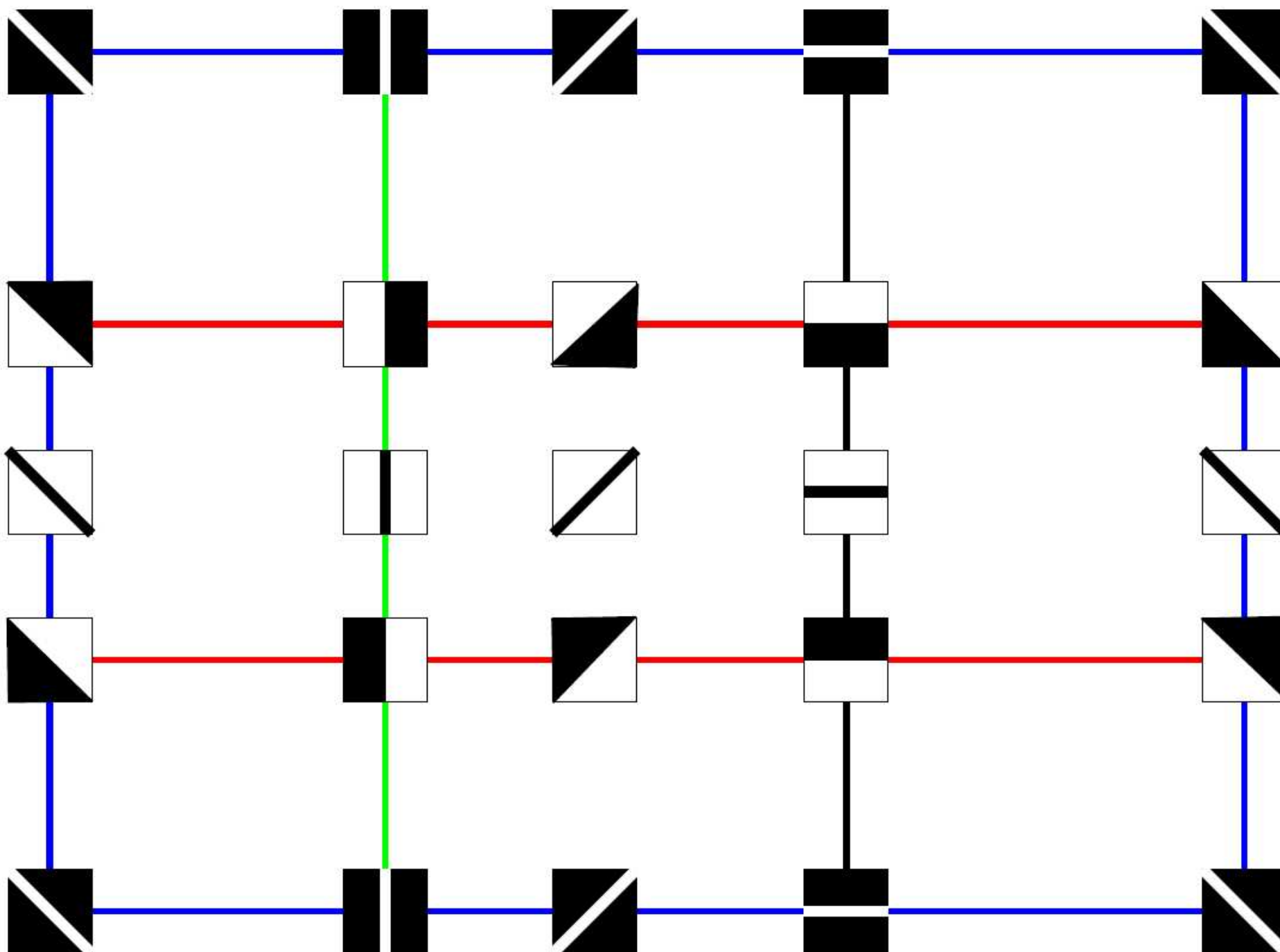
- Most common patch — one colour
- Preprocessing
 1. Remove constant patches
 2. Normalize to norm 1
- Resulting structure: Klein Bottle



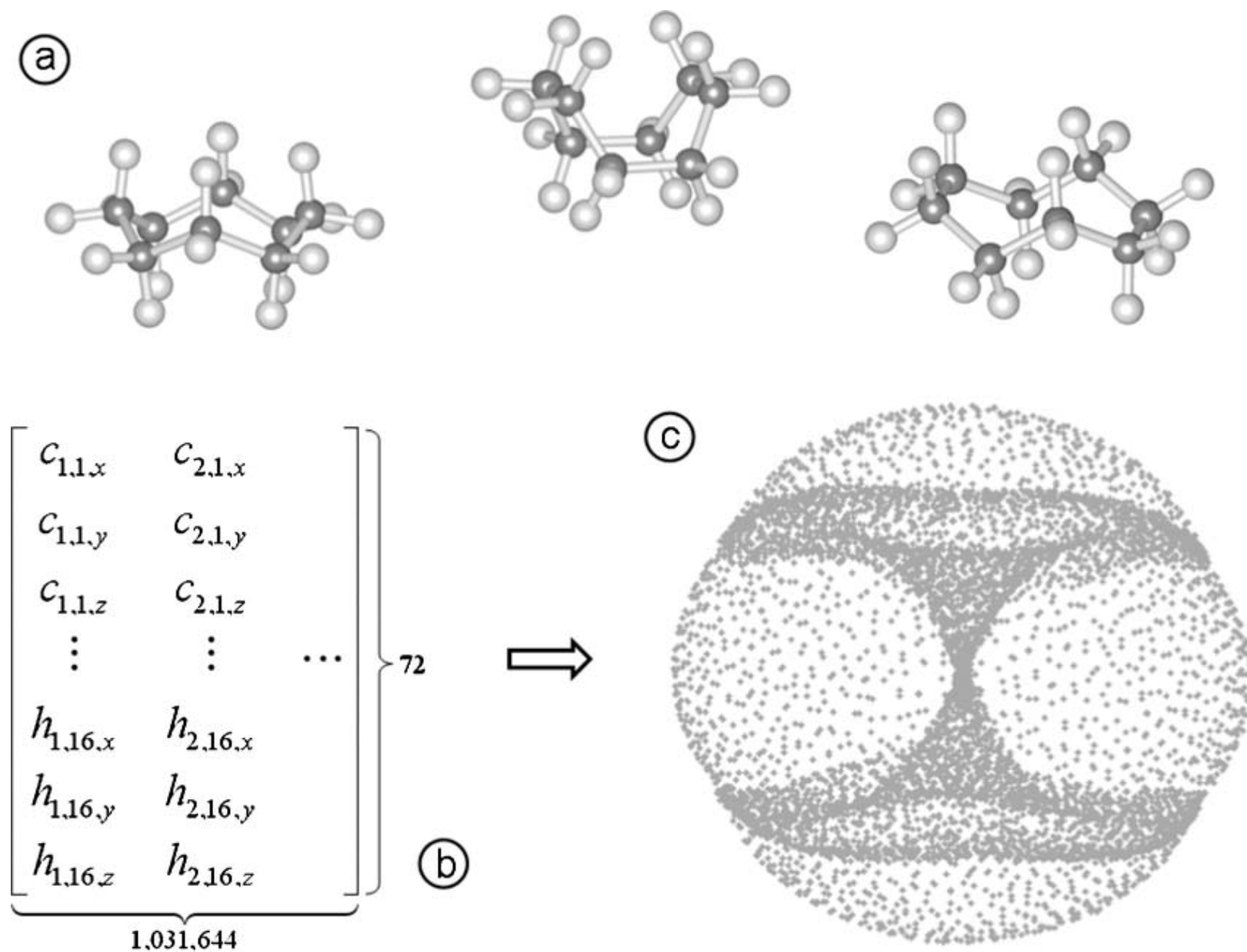
Carlsson, Gunnar, et al. "On the local behavior of spaces of natural images." International journal of computer vision 76.1 (2008): 1-12.

Adams, Henry, and Gunnar Carlsson. "On the nonlinear statistics of range image patches." SIAM Journal on Imaging Sciences 2.1 (2009): 110-117.

Space of Natural Images

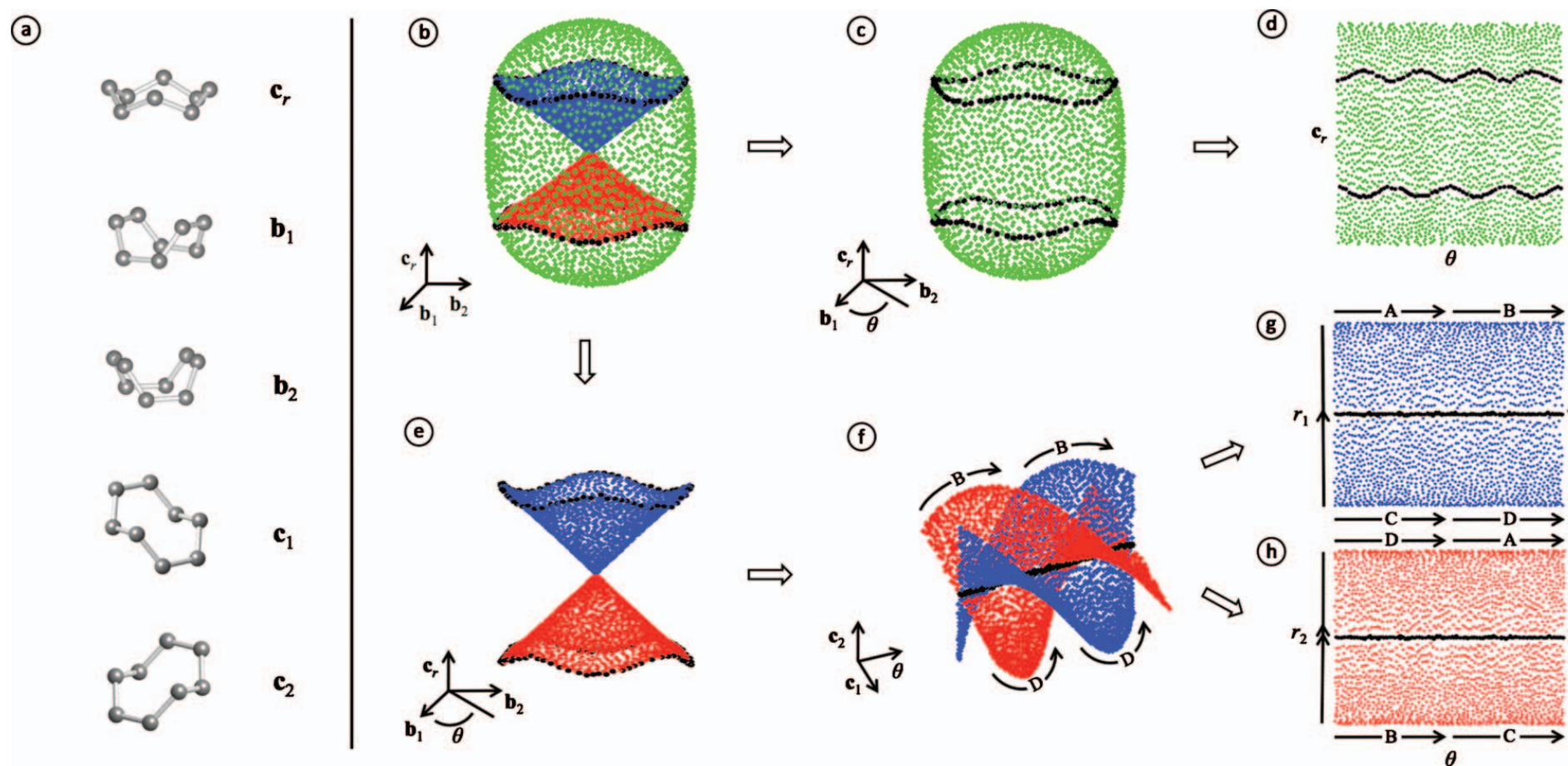
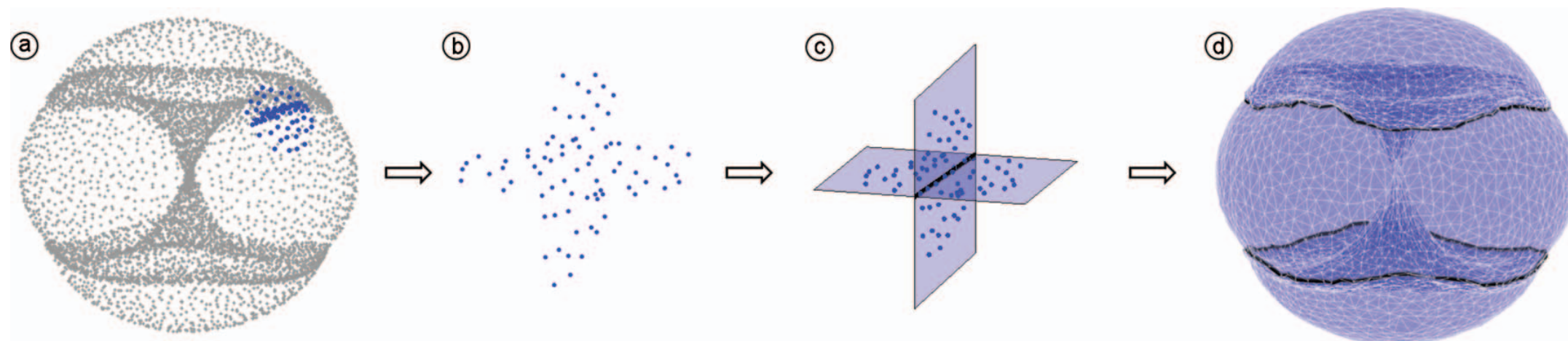


Conformation Space of Cyclooctane



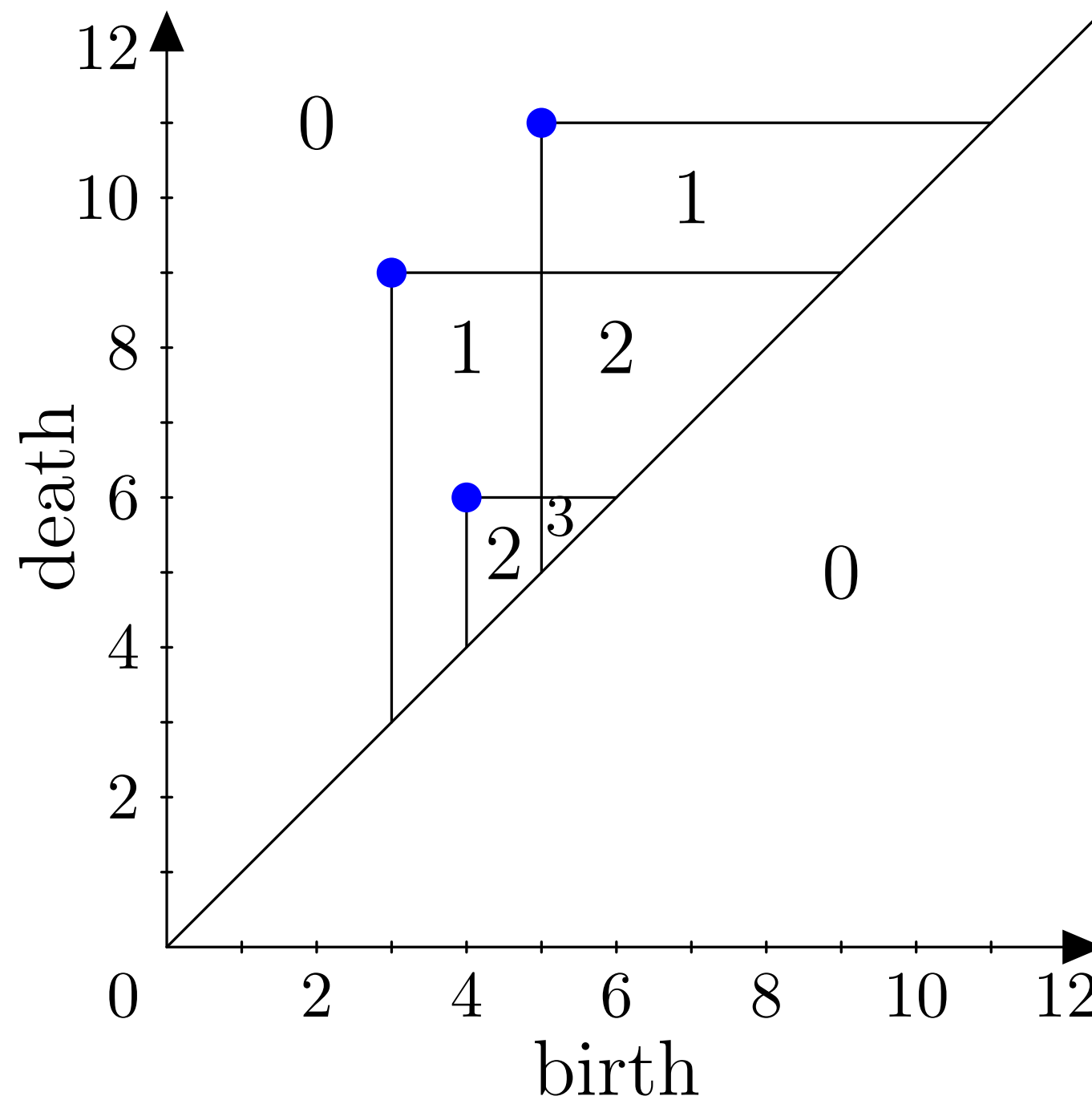
Martin, Shawn, et al. "Topology of cyclo-octane energy landscape." The journal of chemical physics 132.23 (2010): 234115.

Conformation Space of Cyclooctane



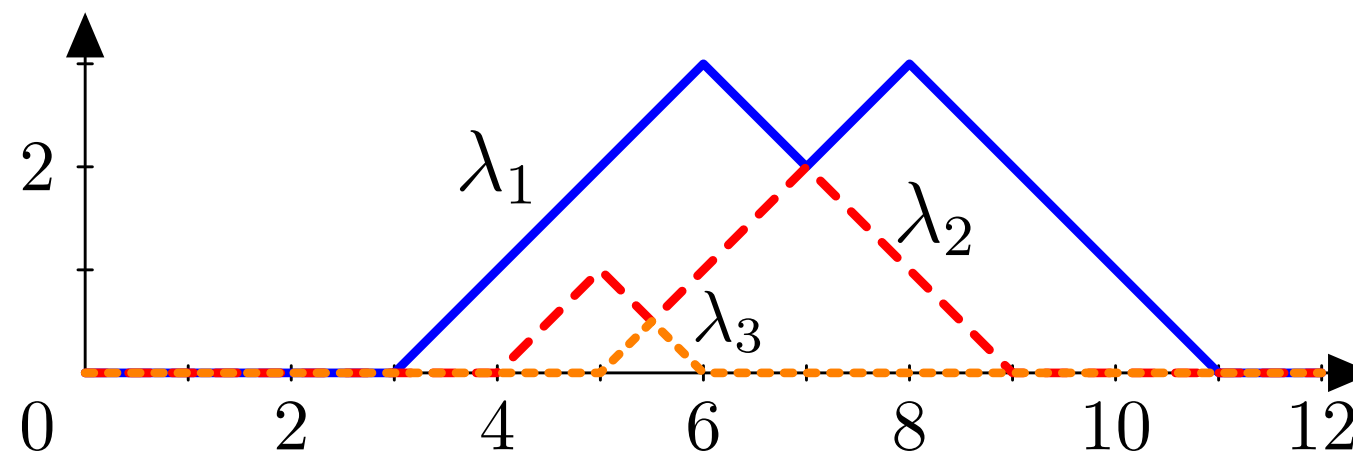
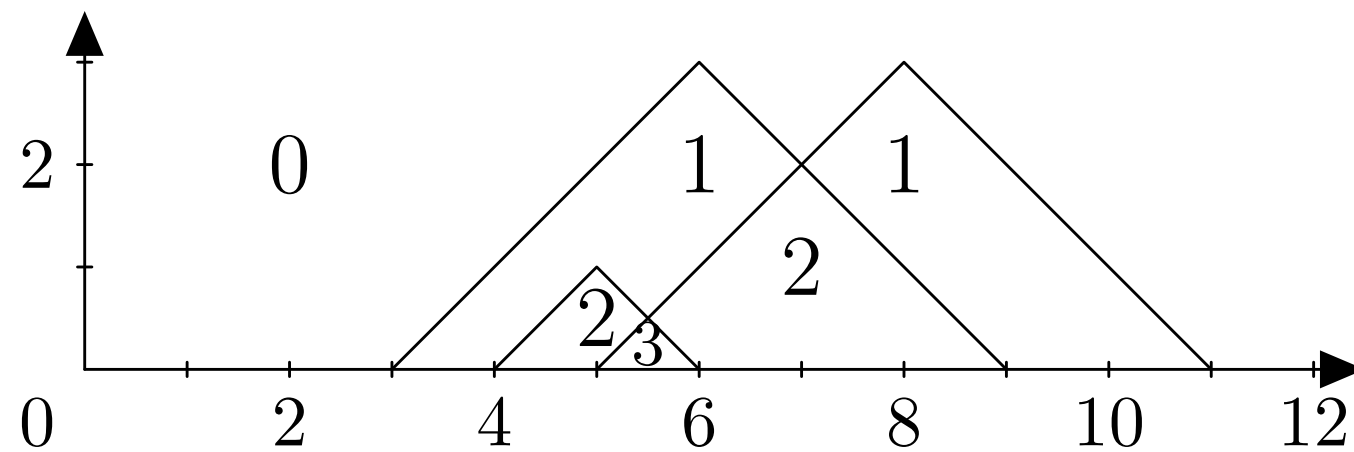
Functionals on Persistence Diagrams

Rank Function

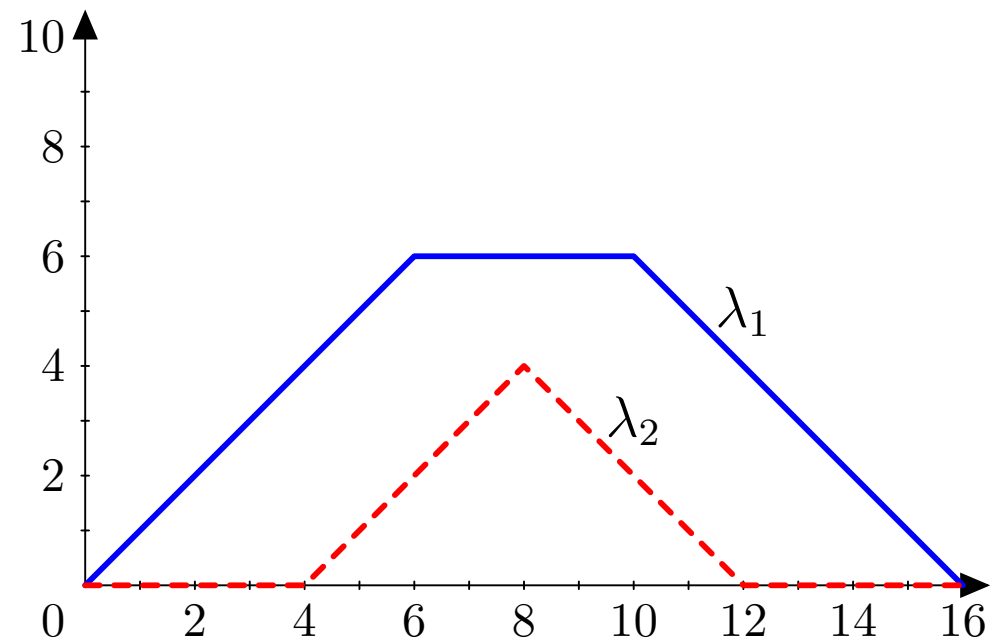
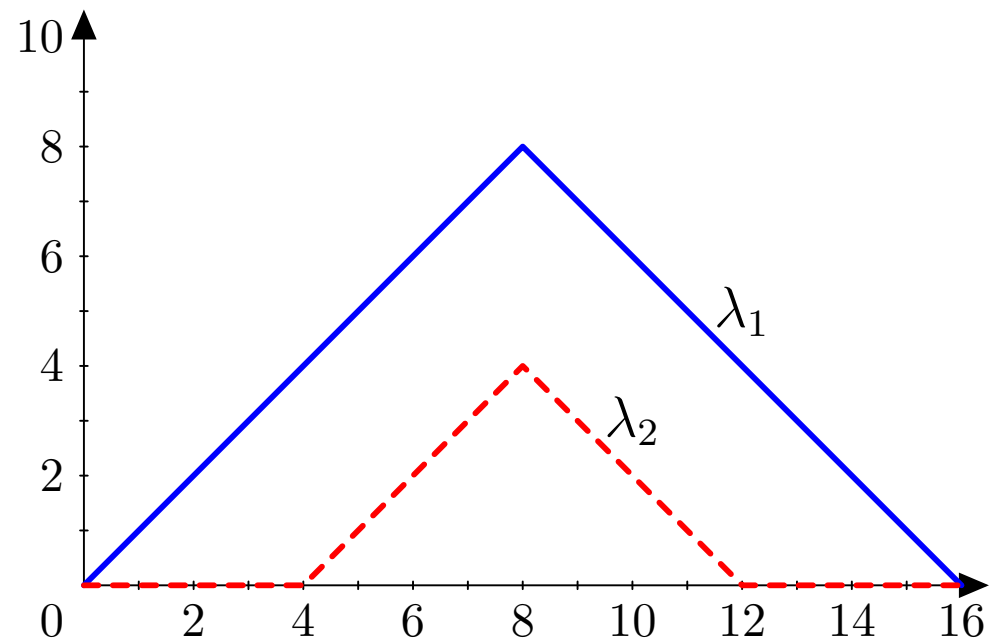
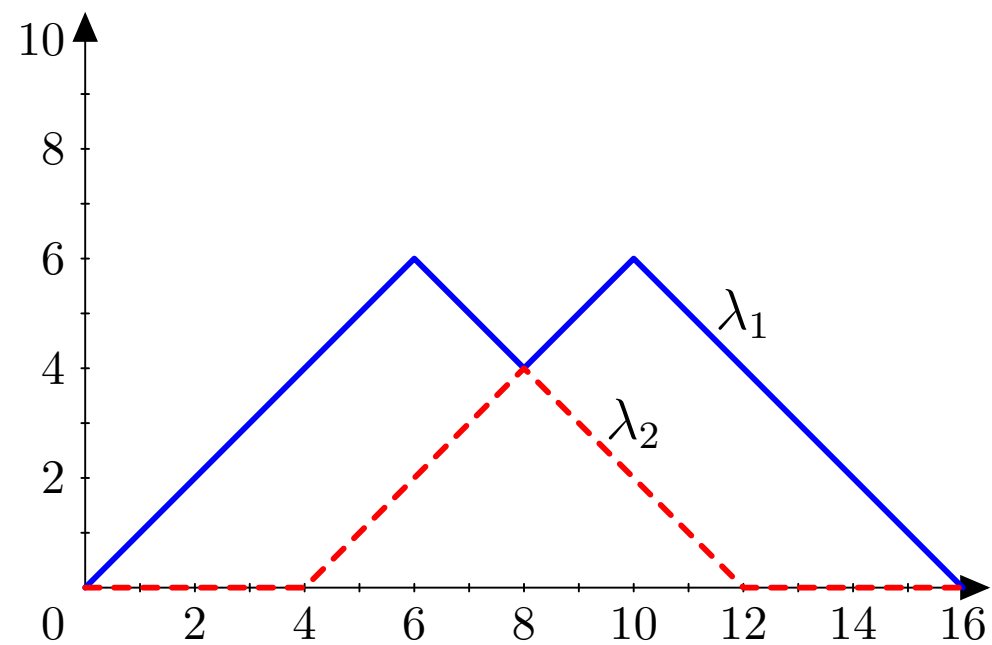
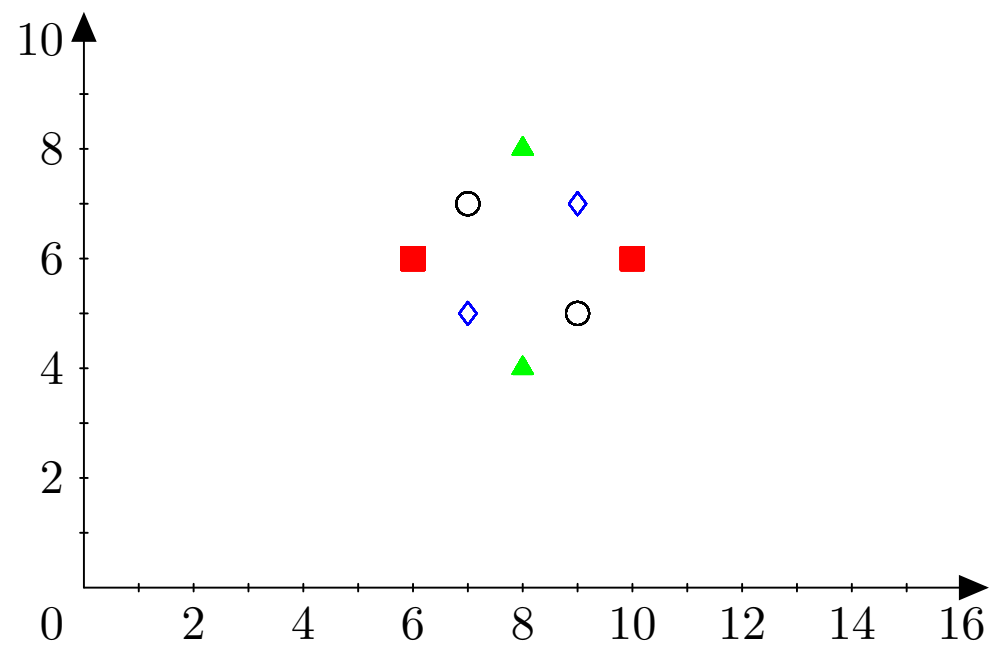


Bubenik, Peter. "Statistical topological data analysis using persistence landscapes." The Journal of Machine Learning Research 16.1 (2015): 77-102.

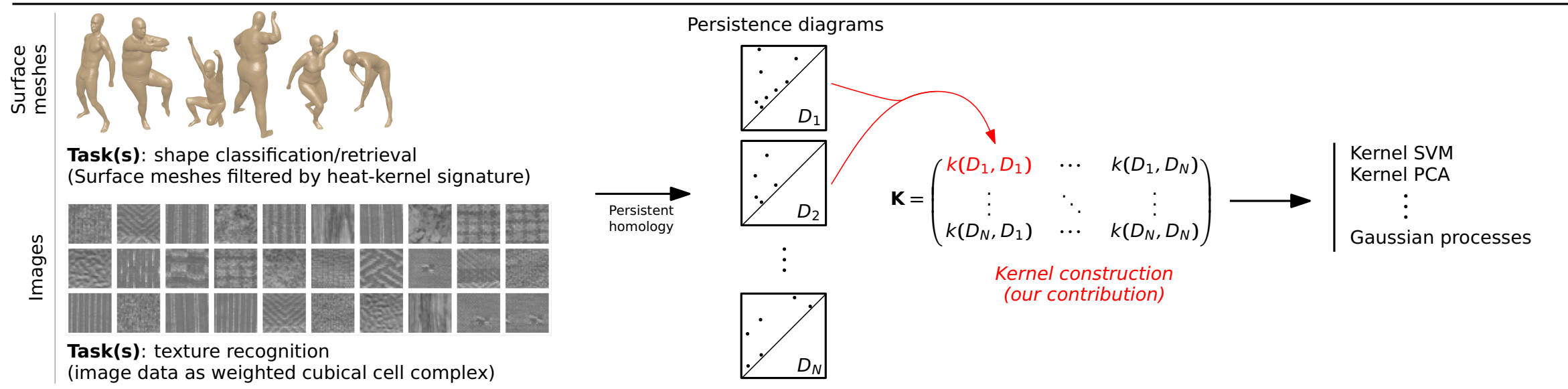
Persistence Landscapes



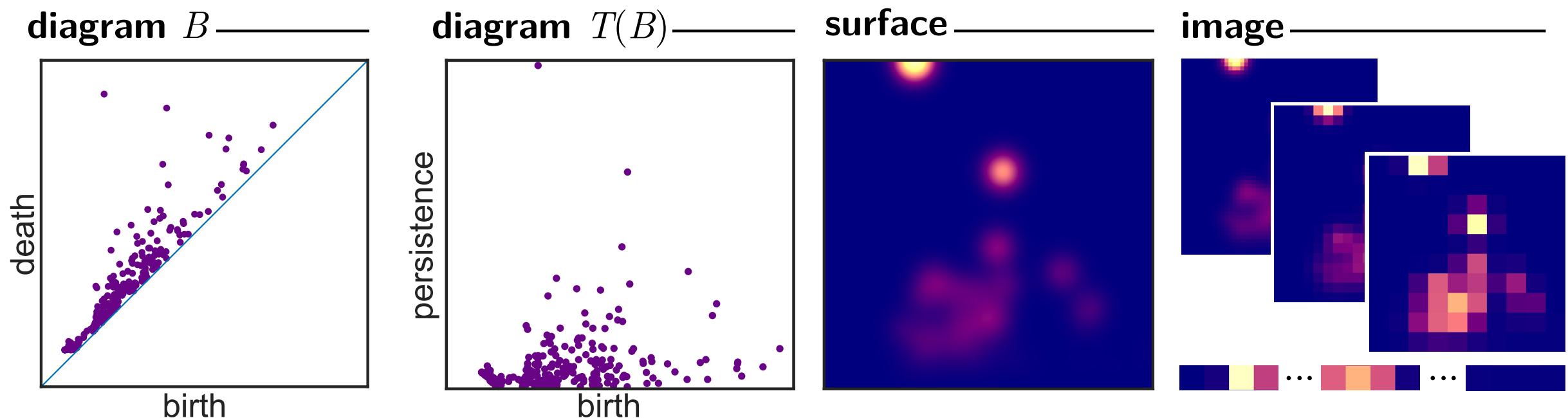
Persistence Landscape



Persistence Images and Kernels



Reininghaus, Jan, et al. "A stable multi-scale kernel for topological machine learning." Proceedings of the IEEE conference on computer vision and pattern recognition. 2015

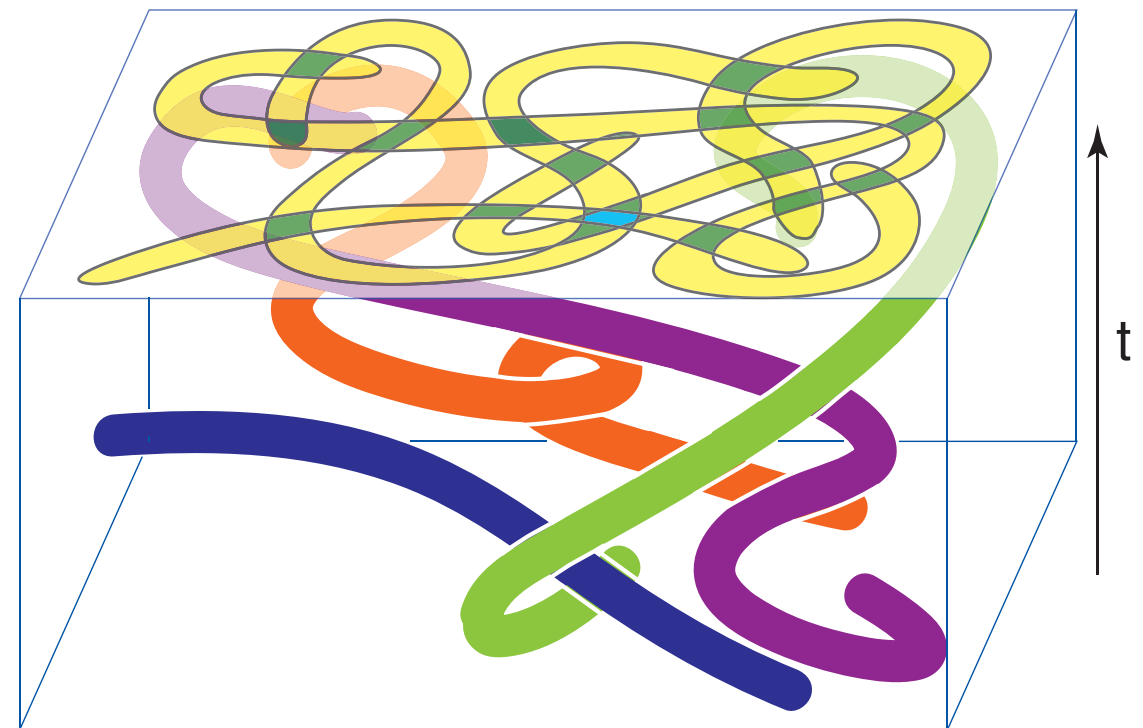
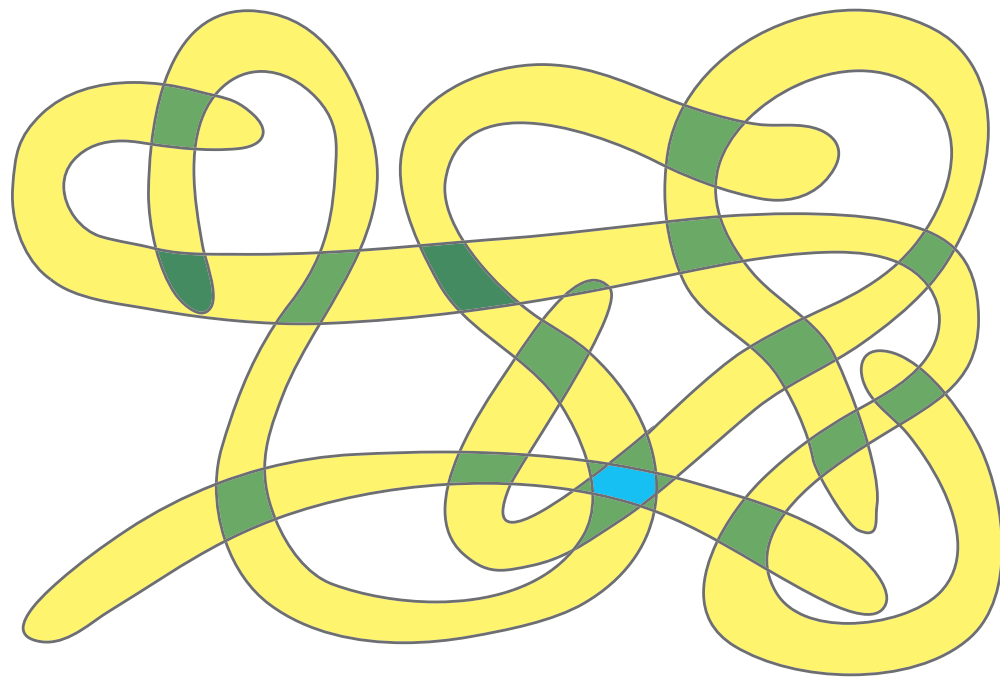


Adams, Henry, et al. "Persistence images: A stable vector representation of persistent homology." The Journal of Machine Learning Research 18.1 (2017): 218-252.

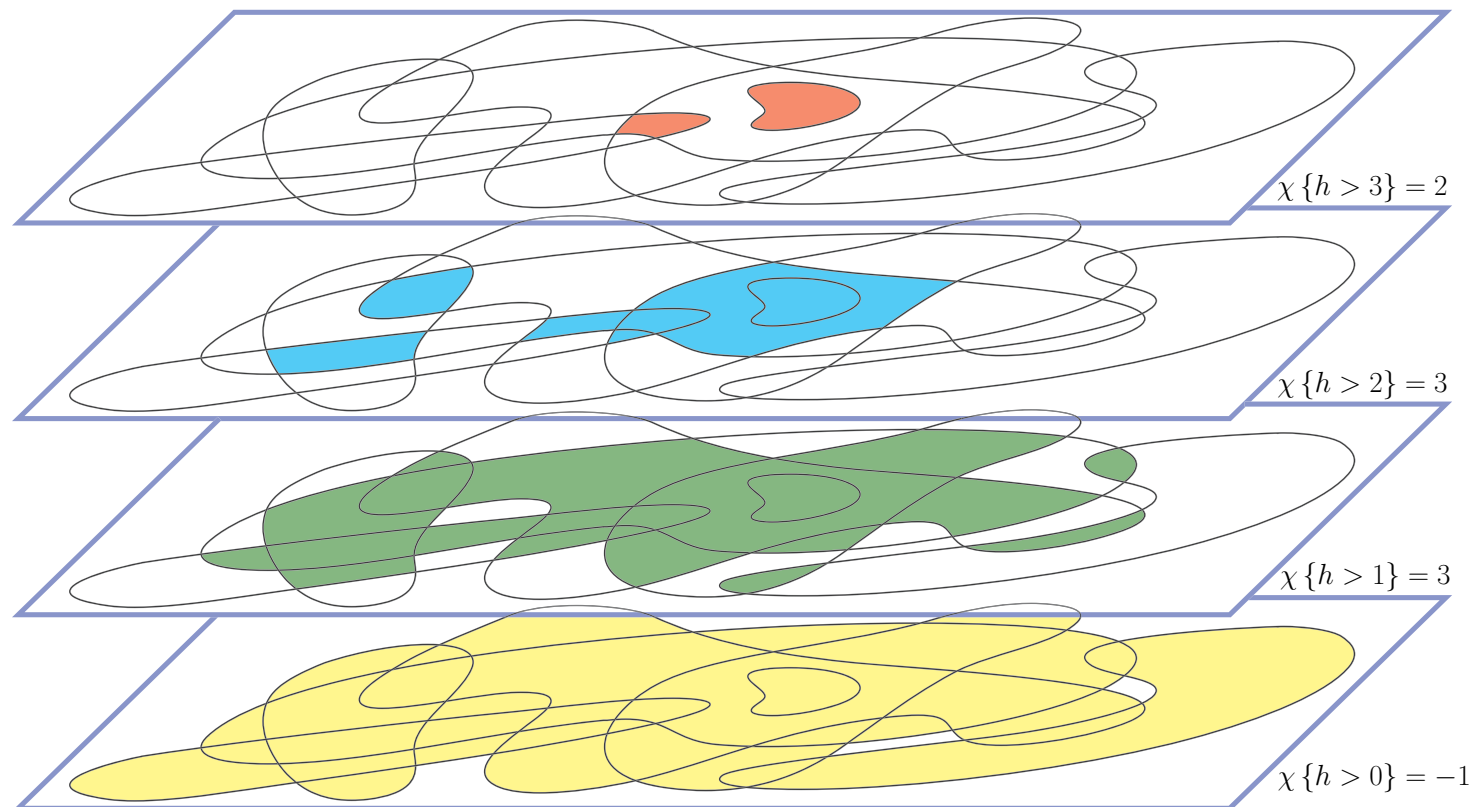
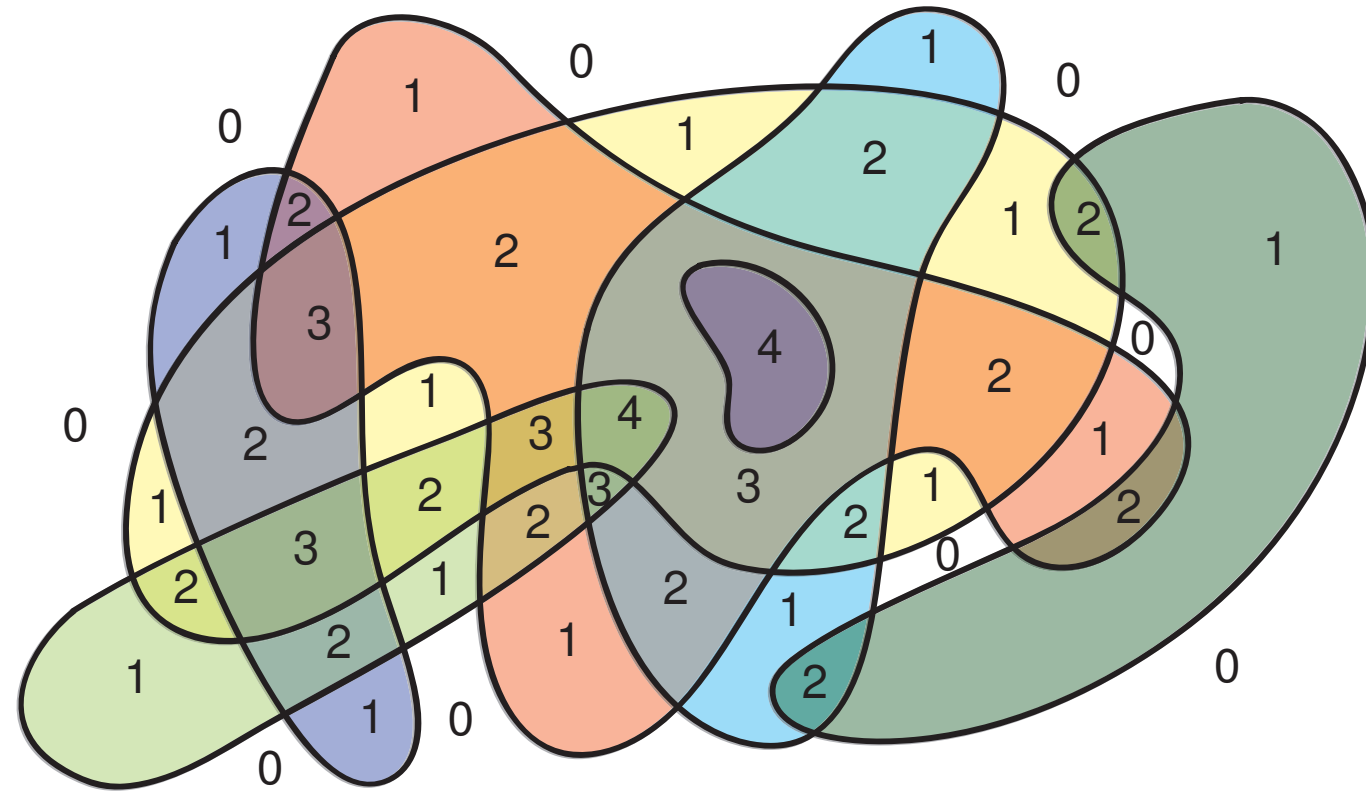
Some Other Applications

Euler Calculus/Integration

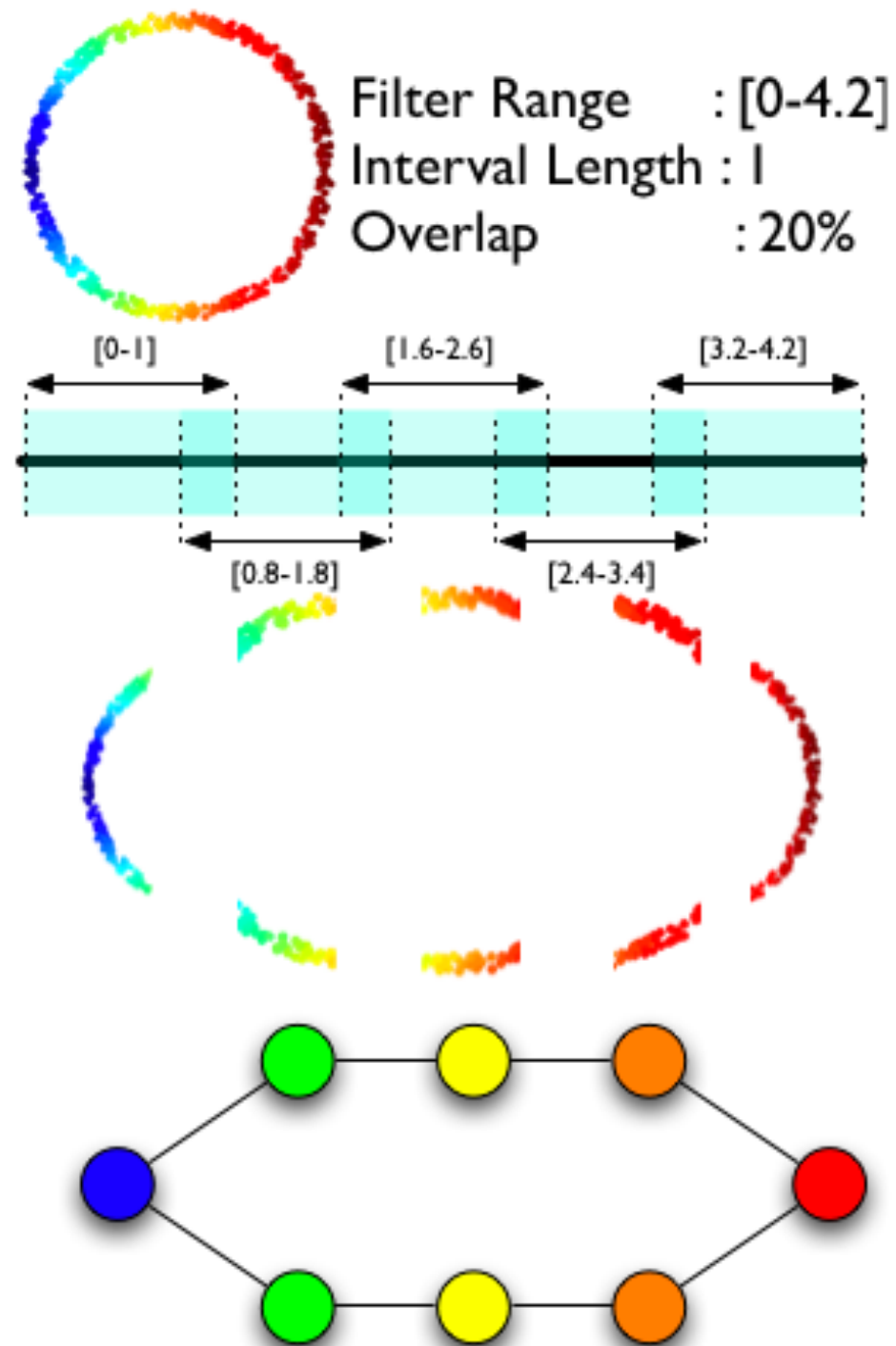
- Euler characteristic is a generalised measure
- Developed by Schapira, Kashiwara, Viro
- Counting trajectories



Euler Calculus/Integration

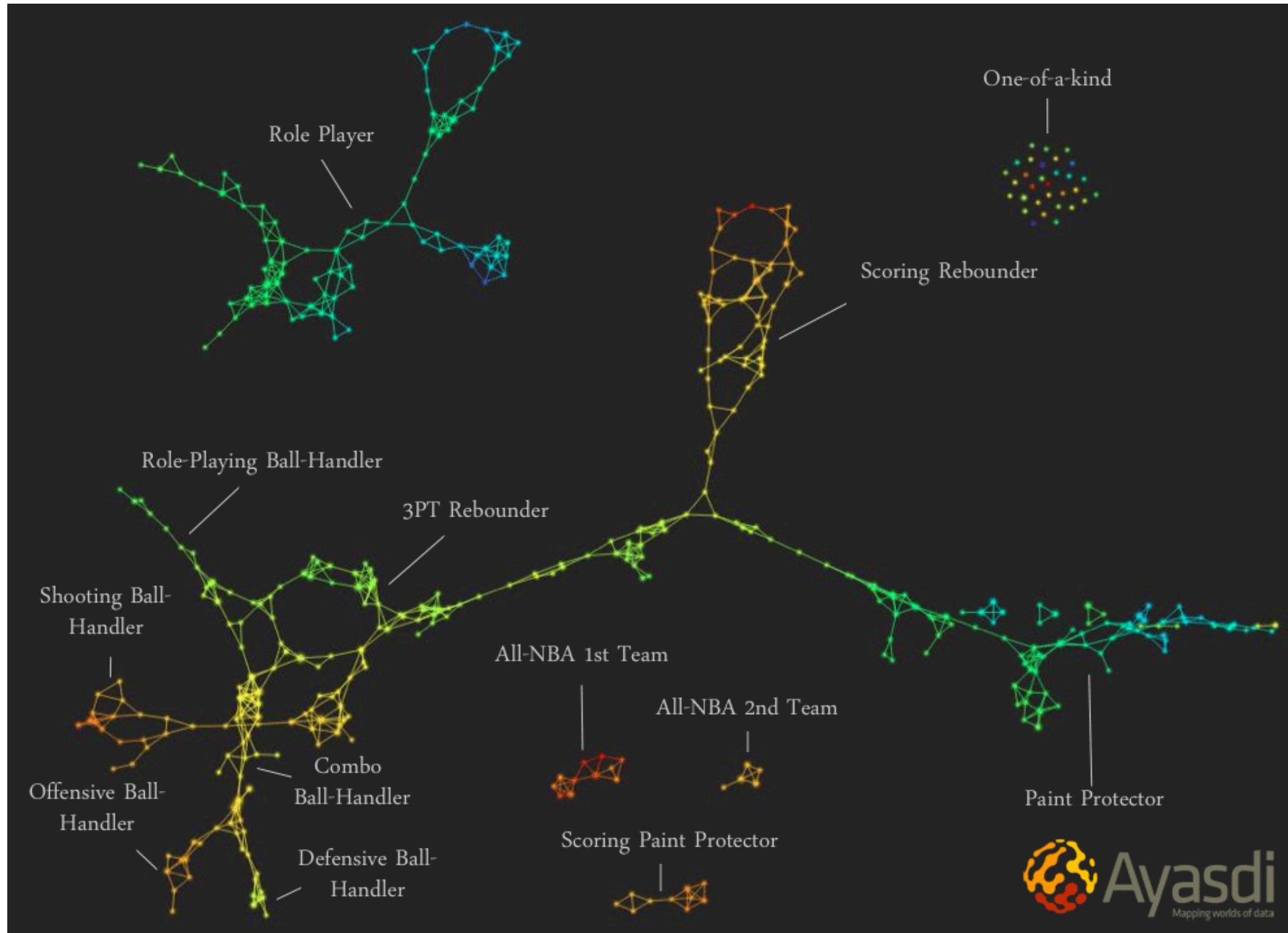


Mapper



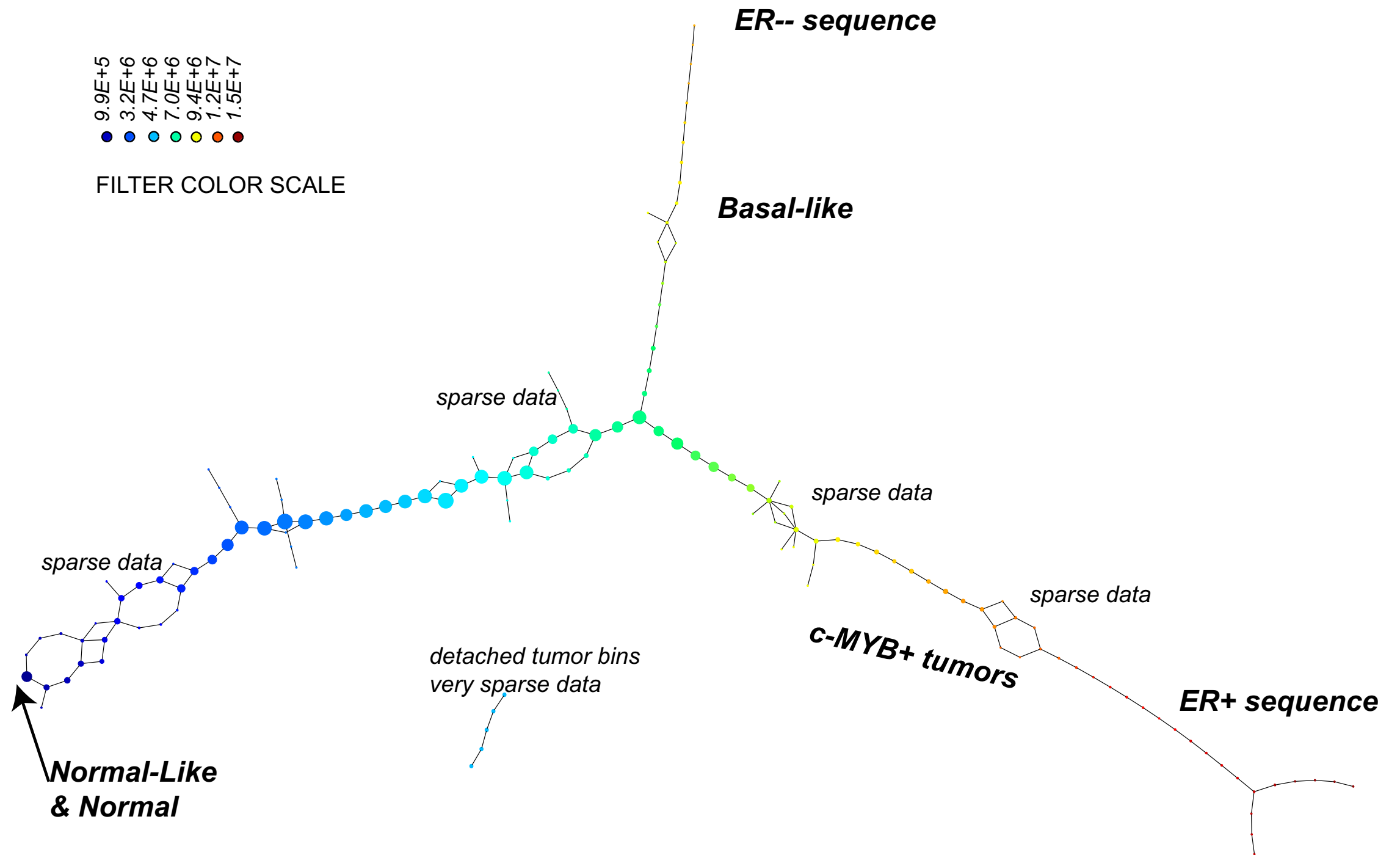
Singh, Gurjeet, Facundo Mémoli, and Gunnar Carlsson. "Mapper: a topological mapping tool for point cloud data." Eurographics symposium on point-based graphics. Vol. 102. 1991.

Mapper



Wired: Analytics Reveal 13 New Basketball Positions (<https://www.wired.com/2012/04/analytics-basketball/>)

Mapper



Nicolau, Monica, Arnold J. Levine, and Gunnar Carlsson. "Topology based data analysis identifies a subgroup of breast cancers with a unique mutational profile and excellent survival." *Proceedings of the National Academy of Sciences* 108.17 (2011): 7265-7270.

Cohomological Coordinates

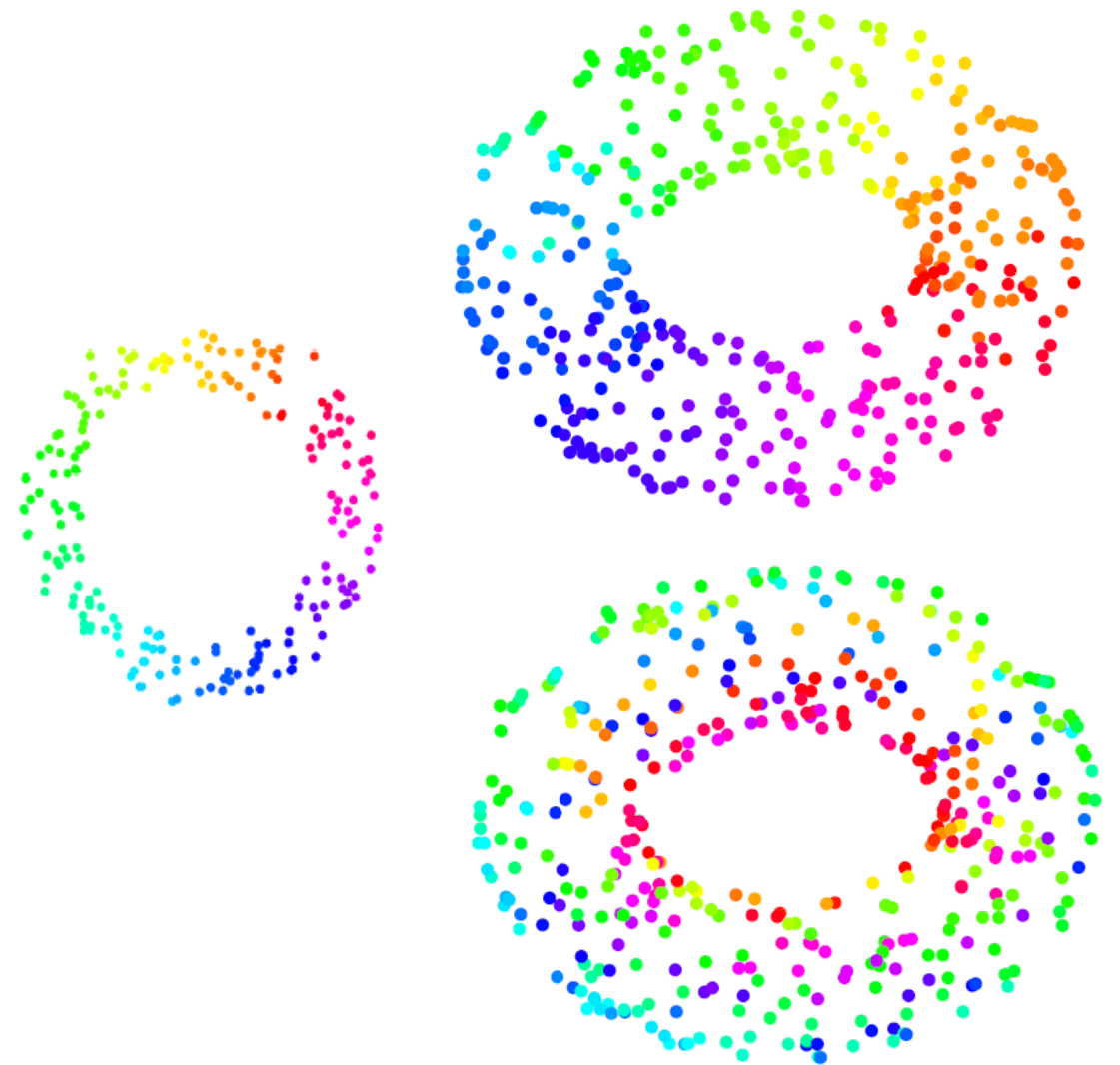
Circle-valued functions:

$$f : \mathbb{X} \rightarrow S^1$$

Fundamental equation

$$[\mathbb{X}, S^1] \cong H^1(\mathbb{X}, \mathbb{Z})$$

This gives an intrinsic
parameterization of a circle

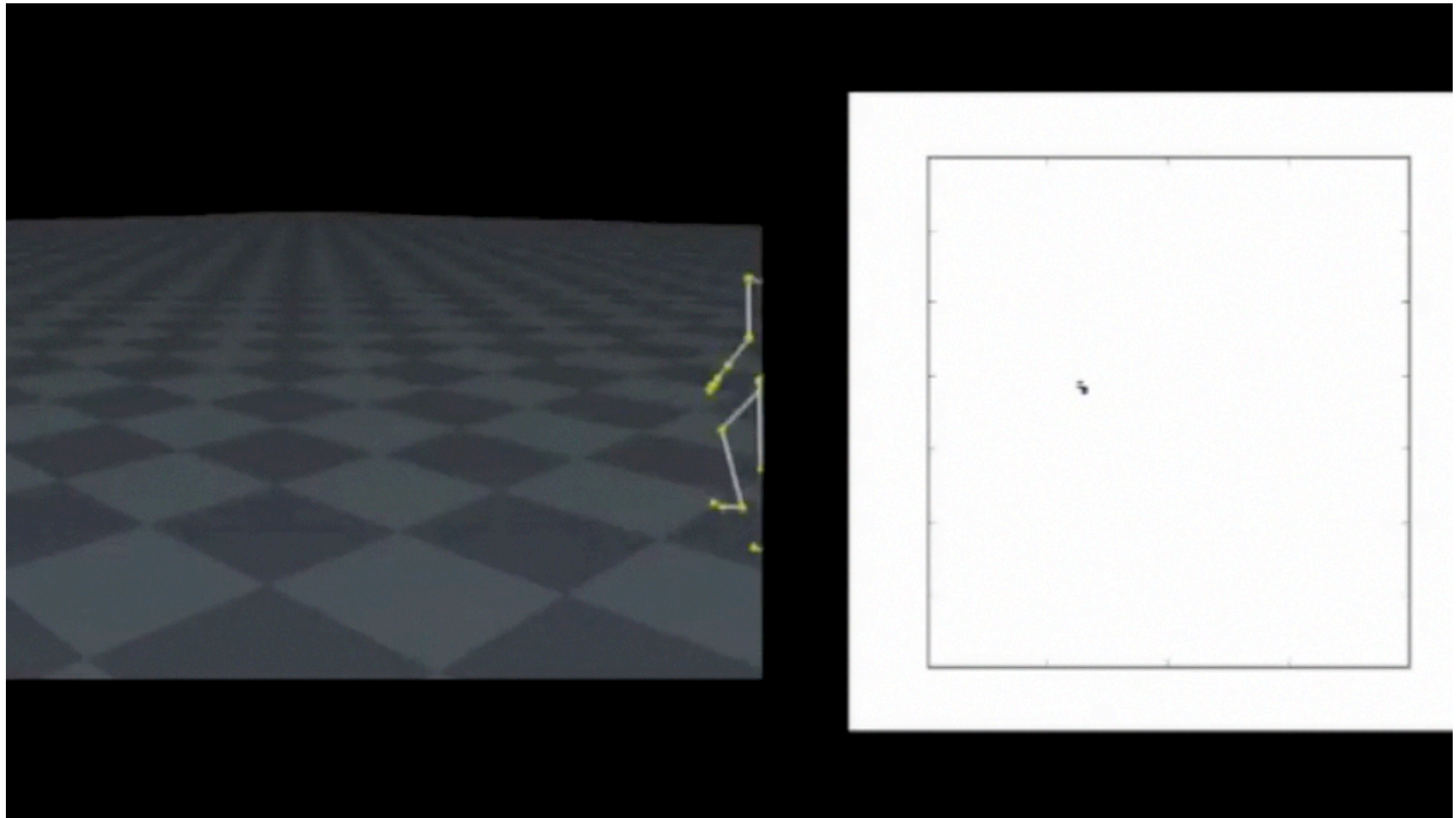


Cohomological Coordinates

Recurrent Behaviour

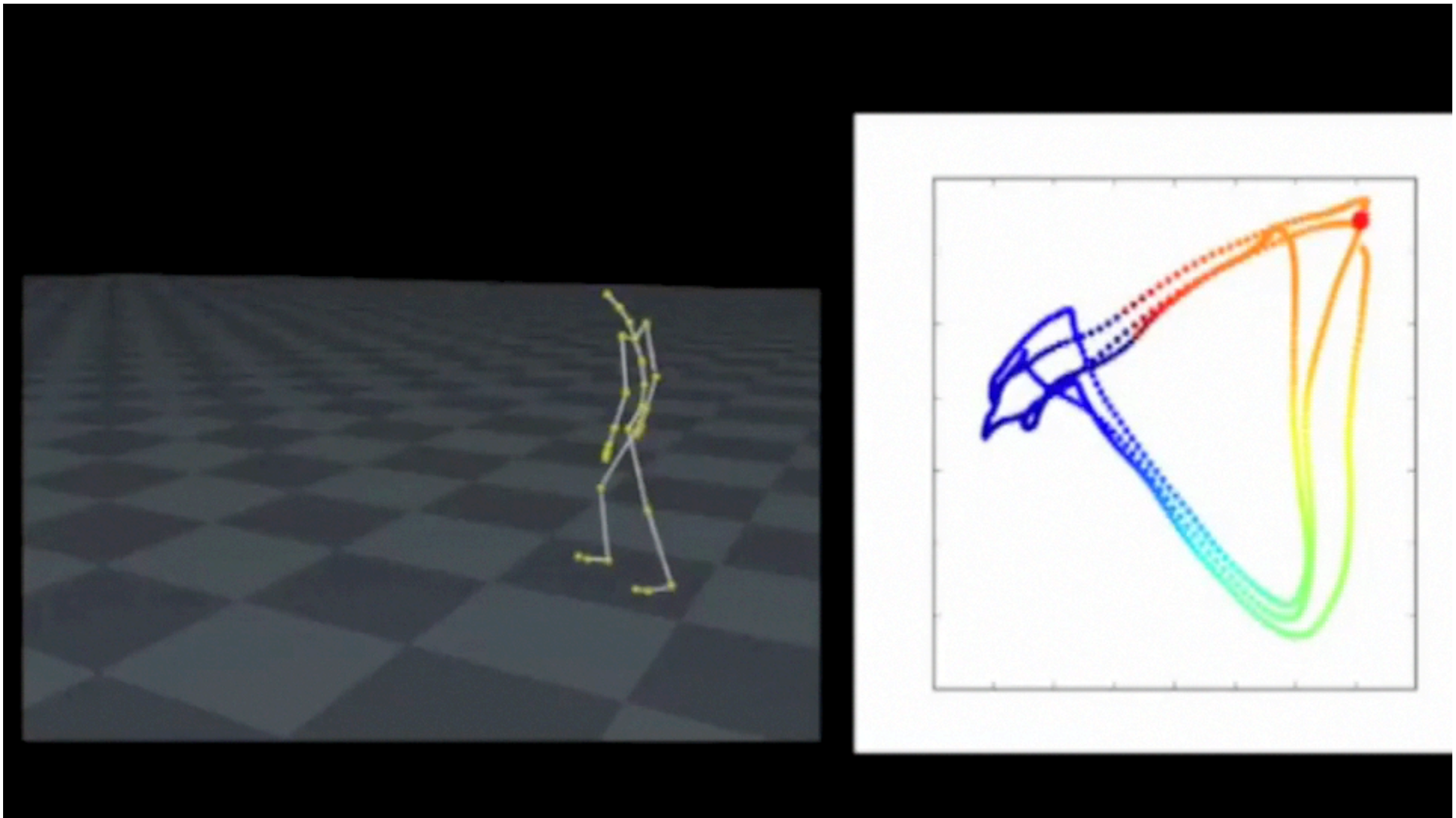


Cohomological Coordinates

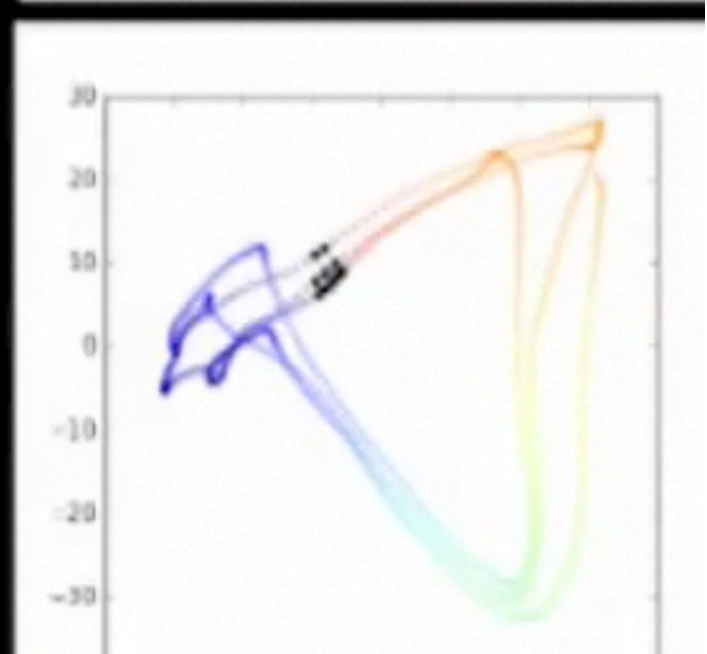
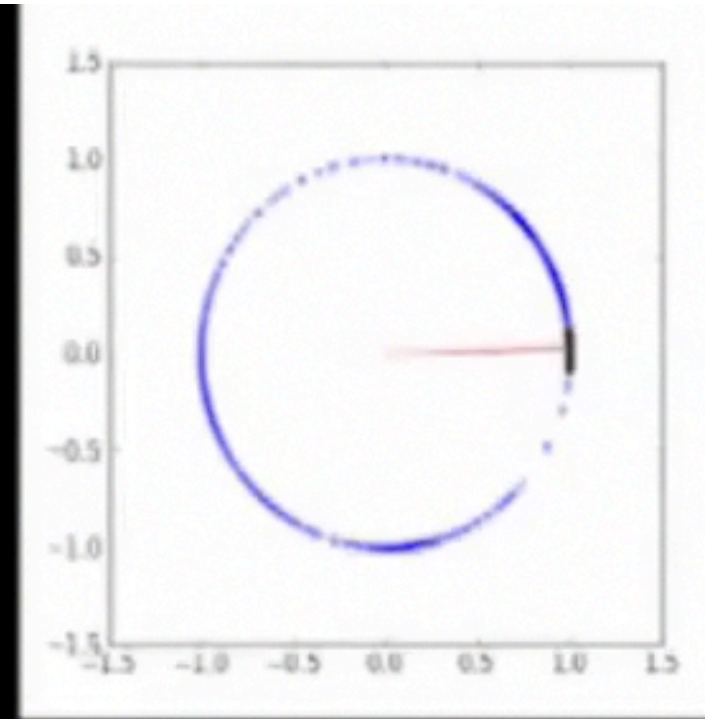
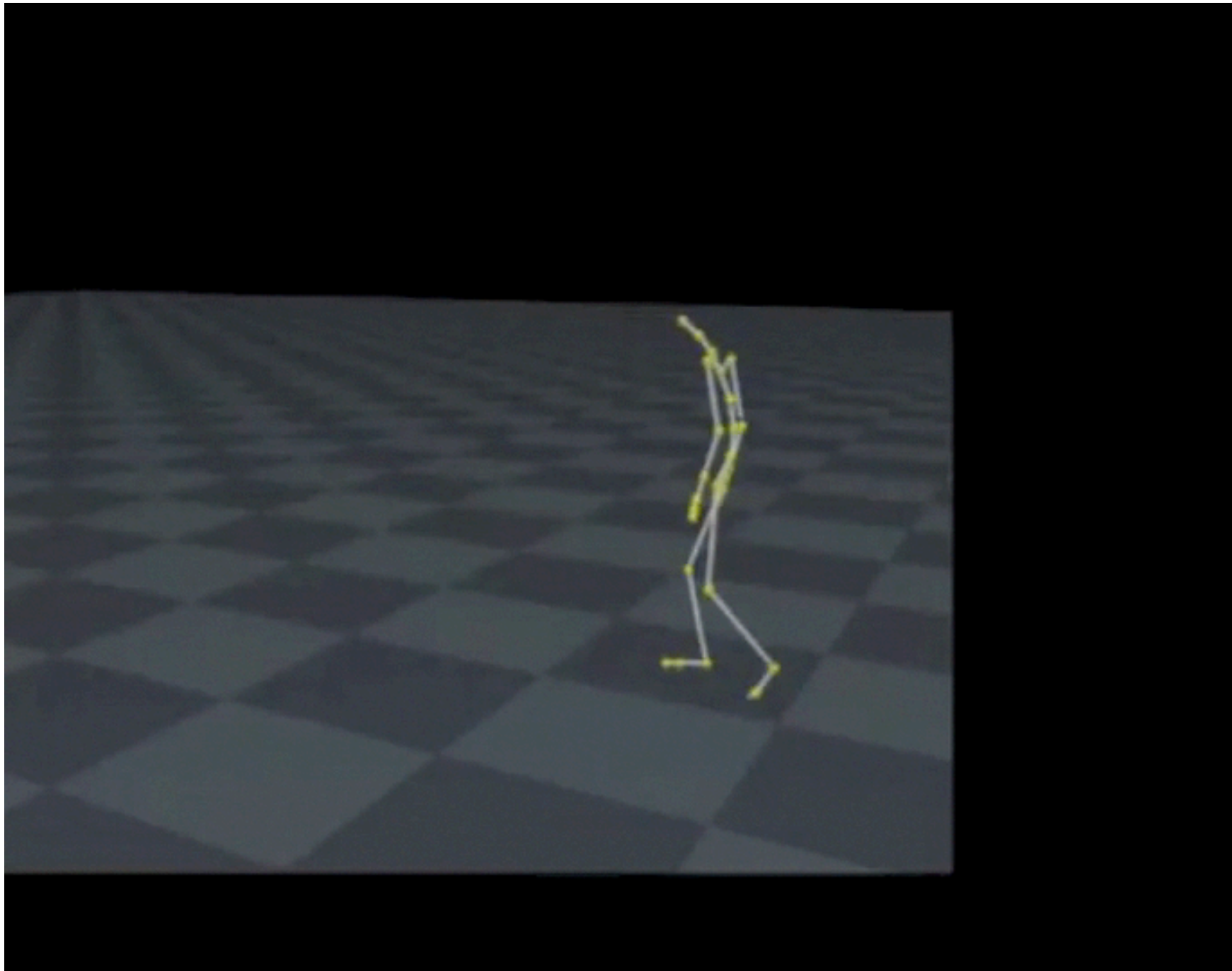


see <https://www.youtube.com/watch?v=NGQ-M2gdibQ>

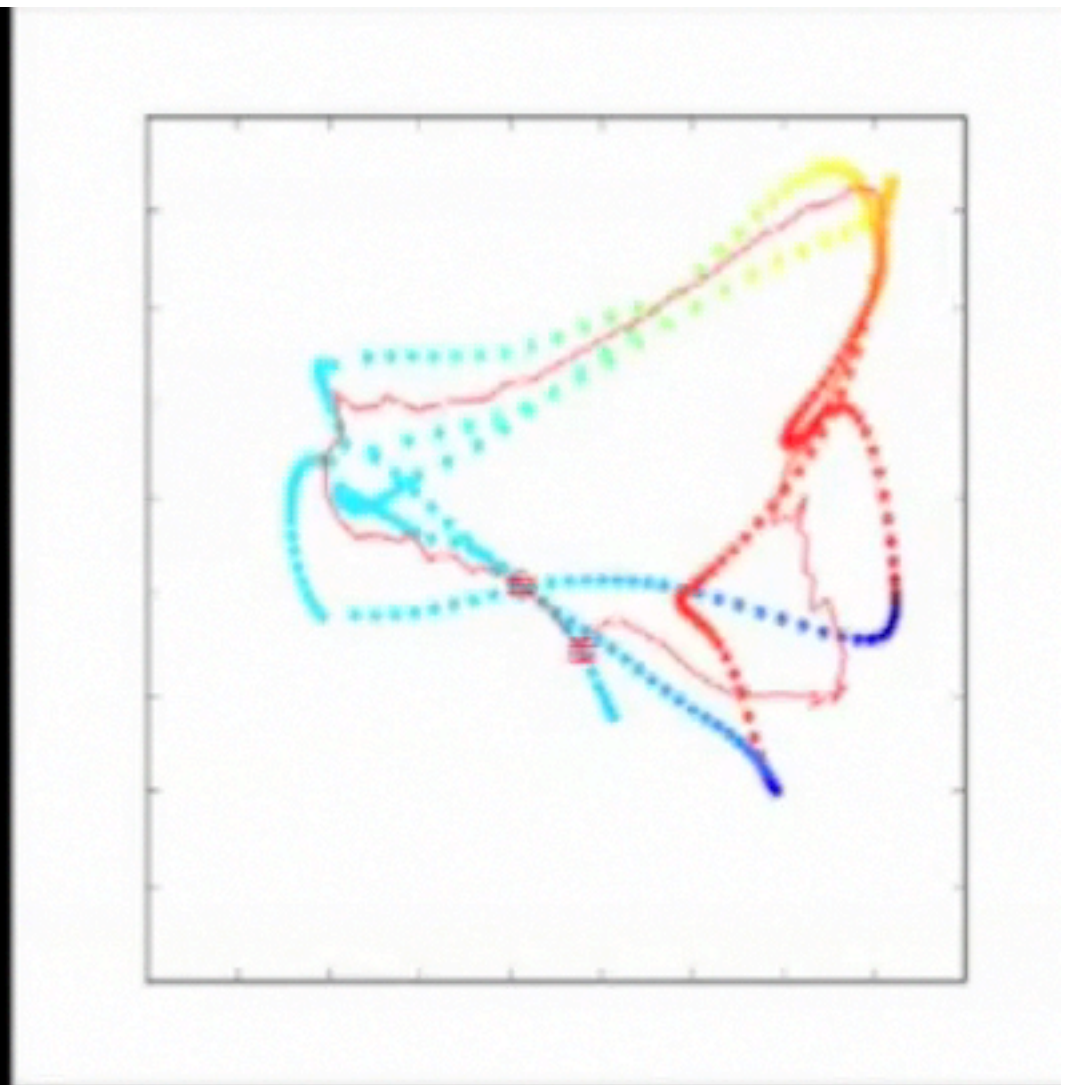
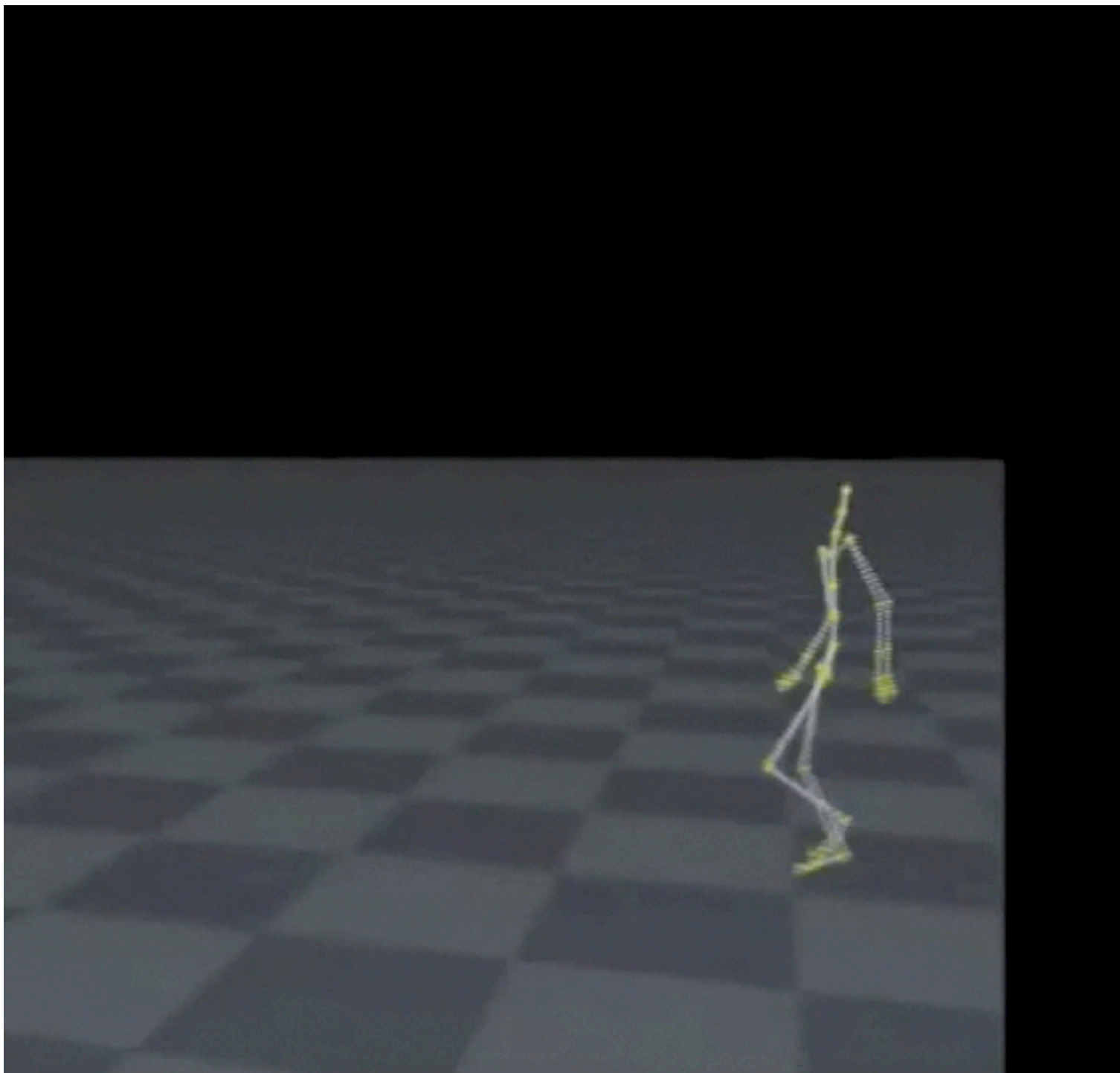
Cohomological Coordinates



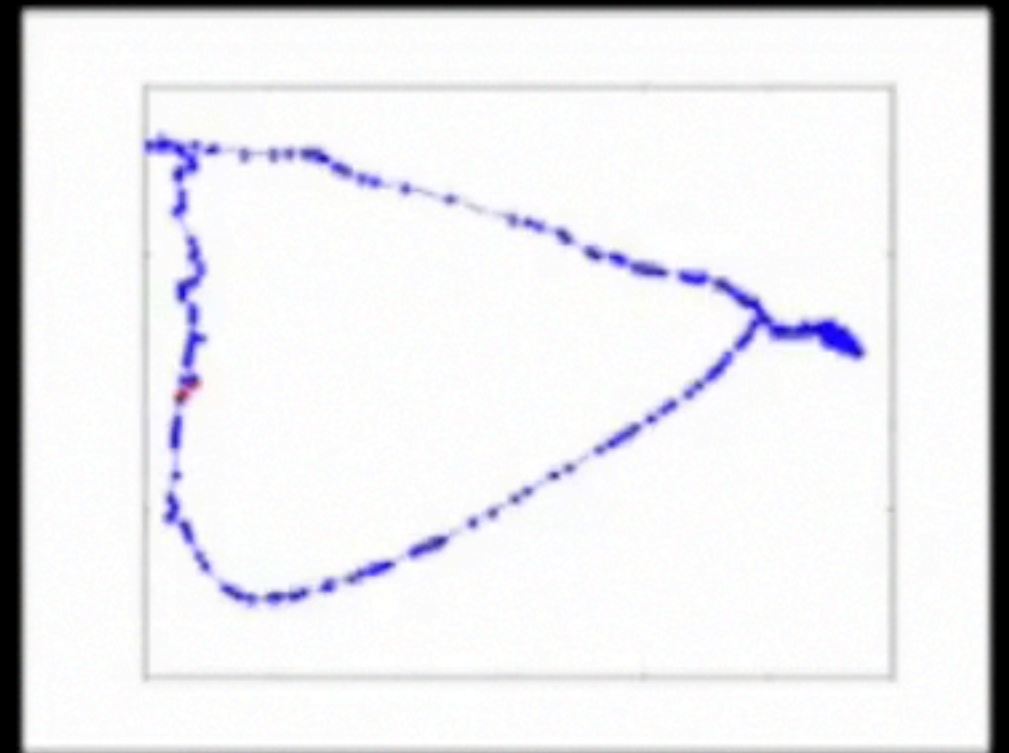
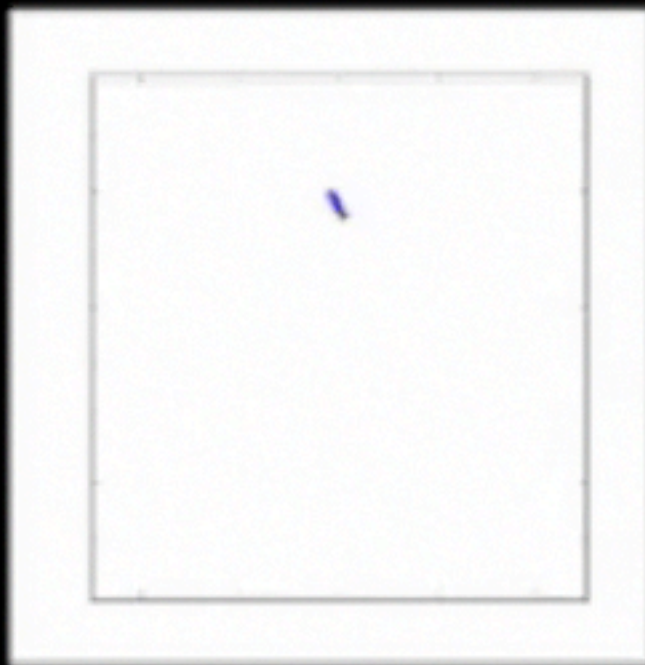
Cohomological Coordinates



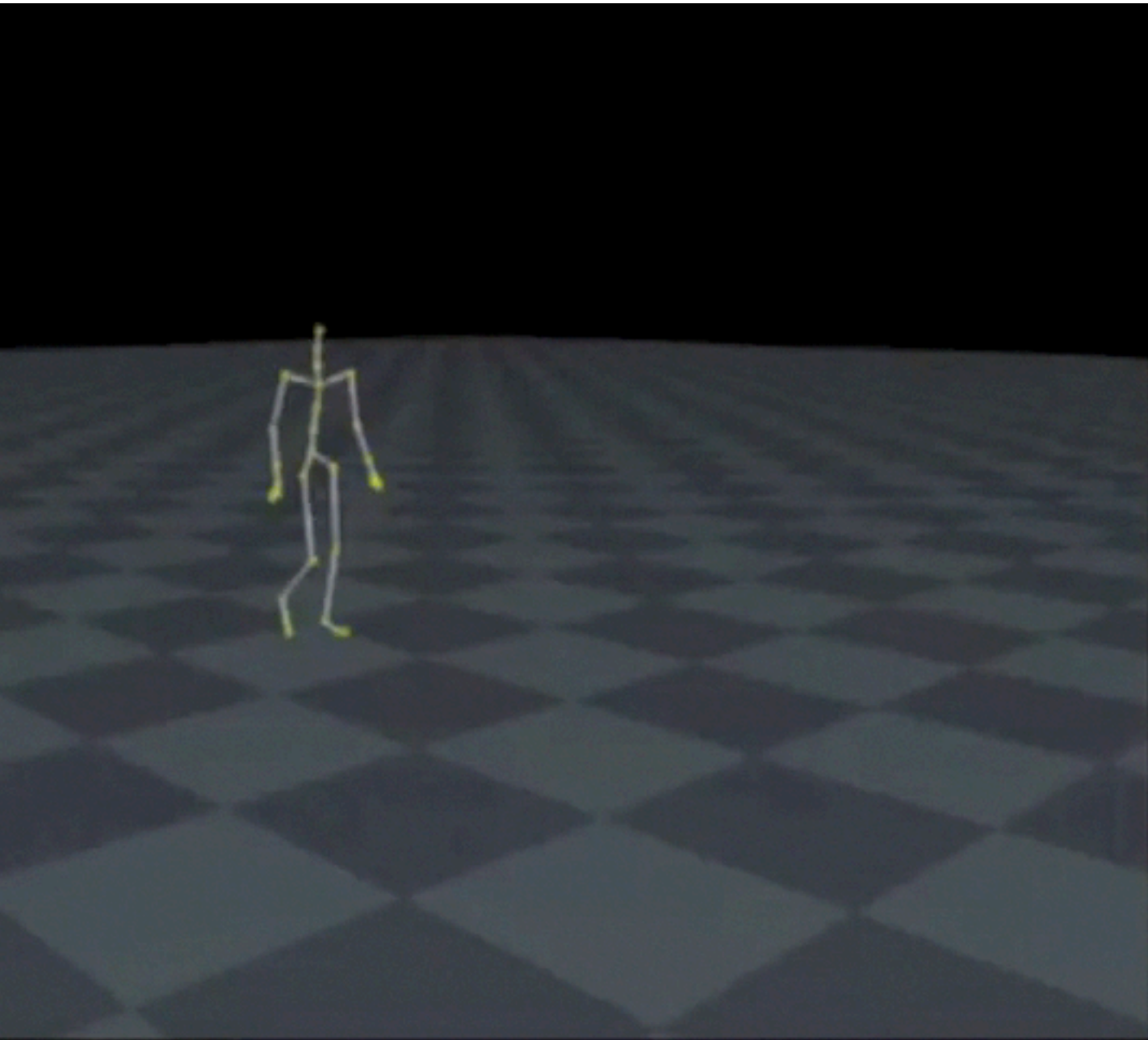
Cohomological Coordinates



Cohomological Coordinates

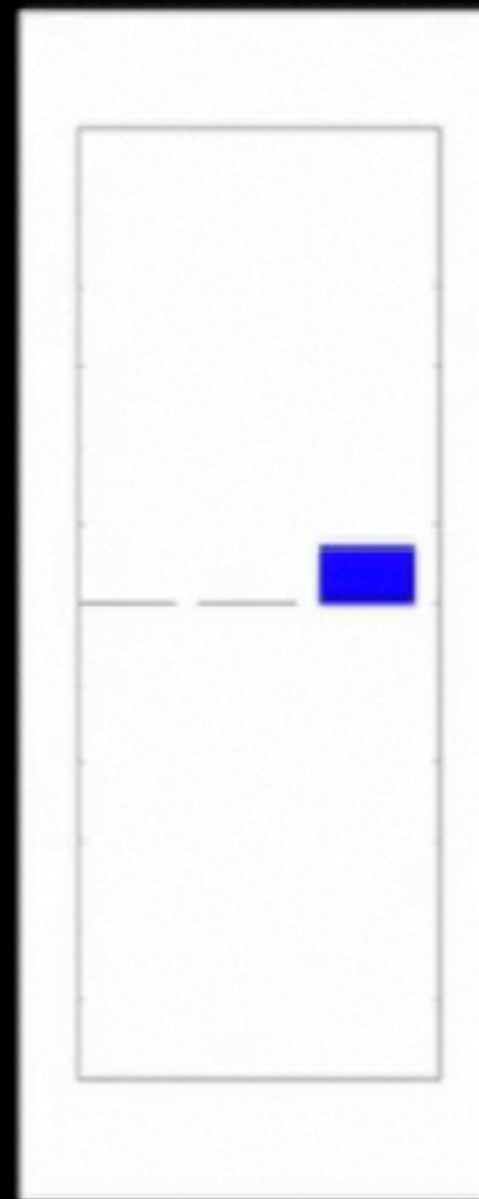


Cohomological Coordinates



eps = 42.6

=100.0



Cohomological Coordinates

