LTCC Basic Course

Title: An introduction to the finite classical groups

Basic Details:

- Core Audience: 2nd/3rdyr pure; (1st year depending on background)
- Course Format (**Extended**: 5 x 2hr lectures)

Course Description:

- Keywords: finite field, vector space, matrix group, quadratic form, sesquilinear form, isometry, isometry group, permutation group.

- Syllabus**:**

    - The Classification of Finite Simple Groups is one of the most celebrated theorems in mathematics. It states that simple groups fall into a number of natural families, one of which is the classical groups. These groups are most easily understood as matrix groups. Our aim will be to define and study these groups using linear algebra and the theory of permutation groups.
    - We will discuss background on permutation groups, fields and vector spaces.
    - We will move on to projective space and linear groups, particularly the family PSL(n,q).
    - We will discuss quadratic and sesquilinear forms, isometries and Witt's lemma.
    - This will allow us to define the other classical families: the symplectic, orthogonal and unitary groups. We will prove some basic properties of these families.

- Recommended reading:

  This course is a condensed version of a course that was taught at the University of Costa Rica in 2014. Online notes available https://nickpgill.github.io/2014/03/11/topics-in- group-theory/

In addition, the following sources are all interesting and relevant.

- *Classical groups* by Peter Cameron. These are online notes available https://webspace.maths.qmul.ac.uk/p.j.cameron/class_gps/
- *The geometry of the classical groups* by Donald Taylor
- *The subgroup structure of the finite classical groups* by Kleidman and Liebeck
- *Permutation groups* by Dixon and Mortimer

- Prerequisites**:**

  I assume that you have done a basic course in group theory and are familiar with the statements of the isomorphism theorems, Lagrange's theorem, Sylow's

theorems and the concept of a group action. I also assume that you have seen a definition of the sign of a permutation, and have met the symmetric group, $\mathrm{Sym}(\Omega)$, and the alternating group, $\mathrm{Alt}(\Omega)$, for a set Ω. Finally, you need to be familiar with some basic facts concerning finite fields.

Format:

I anticipate that there will be 4 discussion/ problem sheets. Lecture notes will be provided after each lecture. These will contain enough information to allow students to complete the course.

Lecturer Details:

- Lecturer: Nick Gill
- Lecturer home institution: The Open University
- Lecturer e-mail: nick.gill@open.ac.uk

Please note that I work only two days per week for the Open University. I will inform students of my work pattern when the lectures start so they know when they can reasonably expect to reply to their communications.