LTCC Advanced Course

Title: Mathematical Methods for Quantum Mechanics

Basic Details:

- Core Audience: Applied and pure mathematics students in any year with some familiarity with quantum mechanics, or at least a willingness to accept the basic principles. (We will not have time to provide much in the way of the physical motivation for the theory, but some short, pre-recorded lectures will be available for anyone interested.)

- Course Format: 5 x 2 hours lectures.

Course Description:

Overview: Quantum mechanics is one of the main pillars of mathematical physics. Its importance to modern physics is impossible to overestimate, but it is also extremely important to mathematics, both as a source of applications for many mathematical concepts and as a source of ideas that have motivated many advances in pure mathematics.

After a quick overview of the postulates of quantum mechanics and an initial look at the hydrogen atom, we will discuss spin, addition of angular momentum and Clebsch-Gordan coefficients. We will go on to study a variety of approximation methods and an introduction to scattering theory.

- Keywords: Quantum mechanics, spin, perturbation theory.

- Syllabus:
  The content taught each week will be

  1. Introduction. Position and momentum operators. Schrödinger's equation. The harmonic oscillator. Particle in a central potential. The hydrogen atom.
  2. Creation and annihilation operators. General principles of quantum mechanics. Symmetries. Heisenberg uncertainty principle. Rotations, angular momentum and spin.
  3. Tensor products of representations. Addition of angular momentum. Clebsch-Gordan coefficients. Approximation methods 1.
  4. Approximation methods 2. Perturbation theory. The Wentzel, Kramers, Brillouin (WKB) approximation.
  5. Scattering theory. Resonances.

  - Recommended Reading:

  S. Weinberg. *Lectures on Quantum Mechanics*. Cambridge University Press, 2nd edition 2015. ISBN-13 978-1107111660.

J.J. Sakurai and J. Napolitano. *Modern Quantum Mechanics*. 2nd edition. Addison Wesley 2011. ISBN 13: 978-0-8053-8291-4

E. Merzbacher. *Quantum Mechanics*. John Wiley & Sons, 3rd edition 1997. ISBN-13 978-0471887027

S. J. Gustafson, I. M. Sigal: *Mathematical Concepts of Quantum Mechanics*, Springer Universitext 3rd Edition 2020. ISBN 13: 9783030595616

Format:

- Printed lecture notes/slides will be available. Five problem sheets will be provided, with full worked solutions made available at the end of the course.

Course lecturer:

- Lecturer: Rod Halburd
- Lecturer home institution: University College London
- Lecturer e-mail: r.halburd@ucl.ac.uk