

## LTCC Proposed Course

### **Title: Minimal and Constant Mean Curvature Surfaces (Advanced)**

#### **Basic Details:**

- Core Audience: 2nd and 3rd year pure Maths, 1st year encouraged to attend.
- Course Format (**Extended**: 5 x 2hr lectures)

#### **Course Description:**

- Keywords: Geometric Analysis, Spectral Geometry, Minimal surfaces, (Allen-Cahn equation?), Eigenvalue optimisation
- Syllabus: Minimal and constant mean curvature surfaces are ubiquitous in geometric analysis, appearing as solutions of shape optimisation problems, in parametrisation of singularities, and as solutions to some differential equations found in nature. Their rich structure comes in part from the fact that they can be described in many different ways: variationally, as solutions of differential equations, via characterisations of their curvature or through some energy functional, and more. Nevertheless, they are notoriously elusive, and finding new constructions or existence proofs for these surfaces with a given topology is an active area of research. In this course, we will start by reviewing definitions and examples before studying existence results from two perspectives: shape optimisers for eigenvalues of the Laplacian or the Dirichlet-to-Neumann map and singular limits of phase-transition models. The latter are semilinear elliptic equations including the Allen-Cahn equation, the Ginzburg-Landau model for superconductivity, and the closely related Yang-Mills-Higgs equations from gauge theory.

- Recommended reading:

M. Karpukhin, A. Métras, Laplace and Steklov extremal metrics via  $n$ -harmonic maps. *J. of Geom. Analysis* 32 (2022), article 154. arXiv:2103.15204. DOI 10.1007/s12220-022-00891-6

O. Chodosh. Geometric features of the Allen-Cahn equation. Available: <https://web.stanford.edu/~ochodosh/AllenCahnSummerSchool2019.pdf>

Martin Li, Free boundary minimal surfaces in the unit ball: recent advances and open questions, <https://arxiv.org/abs/1907.05053>

- Additional Optional reading:

First three sections of Mikhail Karpukhin, Robert Kusner, Peter McGrath and Daniel Stern, Embedded Minimal Surfaces in  $S^3$  and  $B^3$  via equivariant eigenvalue optimization, <https://arxiv.org/pdf/2402.13121>

- Prerequisites:  
Familiarity with Riemannian geometry and partial differential equations

**Format:**

- No of discussion/problem sheets: 4 problem sheets
- Electronic lecture notes: Electronic lecture notes provided before classes

**Lecturer Details:**

- Lecturer: Jean Lagacé and Stephen Lynch
- Lecturer home institution: King's College London
- Lecturer e-mail: jean.lagace@kcl.ac.uk