LTCC Advanced Course

Title: The Painlevé equations and the Painlevé property

Basic Details: This course describes how singularity structure of the solutions of an ODE in the complex domain can be used to detect integrable equations. The six Painlevé (differential) equations arise in many applications in a remarkable number of fields. We will study the special properties of these equations as well as their integrable discrete analogues.

- Core Audience: Applied and pure mathematics students
- Course Format: 5 x 2hr lectures
- Prerequisites: Basic complex analysis and group theory. Some familiarity with Hamiltonian systems and asymptotic methods is desirable but not necessary. The discussion of the applications described will be self-contained and no special knowledge is required.

Course Description:

- Keywords: Painlevé property; Painlevé equations; iso-monodromy problems; Riemann-Hilbert problems; discrete Painlevé equations; affine Weyl groups.

- Syllabus:
  The content taught each week will be:
  1. Introductory material. ODEs in the complex domain. The Painlevé property and related tests.
  2. The Painlevé equations. Bäcklund transformations and special solutions. Applications to soliton equations, water waves and random matrix theory.
  3. Iso-monodromy problems and Riemann-Hilbert problems.
  4. Construction of Okamoto's space of initial conditions from blow ups of projective space. The origin of the affine Weyl group structure of these surfaces and of the group of Bäcklund transformations.
  5. The QRT map. Discrete Painlevé equations. Sakai's construction using rational surfaces. Algebraic entropy.

- Recommended Reading:

1. M. J. Ablowitz and P. A. Clakson. Solitons, nonlinear evolution equations and inverse scattering. London Math. Soc. Series 149. *Cambridge University Press, Cambridge,* 1991.
2. A. S. Fokas and A. R. Its, A. A. Kapaev and V. Yu. Novokshenov. Painlevé transcendents. The Riemann-Hilbert approach. Mathematical Surveys and Monographs 128, AMS, Providence RI, 2006.
3. E. L. Ince. Ordinary Differential Equations. *Dover Publications, New York,* 1944.
4. K. Kajiwara, M. Noumi, Y. Yamada. Geometric Aspects of Painlevé equations. J. Phys. A 50, no. 7, 073001. 2017.

Format: Printed lecture notes will be available. Five problem sheets will be provided, with full worked solutions made available at the end of the course.

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