LTCC Basic Course

Title: Symmetry Methods for Differential Equations

Basic Details:

- Course Format (Extended: 5 x 2hr lectures)

Course Description:

There is much interest in the determination of symmetry reductions (sometimes known as similarity reductions) of physically and mathematically interesting partial differential equations. Symmetry reductions are usually obtained either by seeking a solution in a special form or, more generally, by exploiting symmetries of the equation. Such symmetries provide a technique for obtaining exact and special solutions, typically in terms of solutions of ordinary differential equations. There have been considerable developments in the application of symmetry analysis (or group methods), which are highly algorithmic, to partial differential equations.

In this series of lectures I shall discuss various techniques for deriving symmetry reductions, in particular

- the classical Lie group method of infinitesimal transformations which dates back to the work of Sophus Lie in the 19th century,
- the "direct method" due to Clarkson & Kruskal [J. Math. Phys., 30 (1989) 2201–2213], and
- the "nonclassical method" due to Bluman & Cole [J. Math. Mech., 18 (1969) 1025–1042], which is also sometimes known as the "method of conditional symmetries".

Emphasis will be placed on explicit computational algorithms to discover symmetries admitted by differential equations and the construction of reductions and exact solutions arising from these symmetries. I shall give a comparison of these three methods discussing their relative strengths and weaknesses.

All the aforementioned symmetry methods involve the solution of an overdetermined system of partial differential equations. An introduction will also be given to the theory of Differential Grobner Bases which provide a "triangularization" of the system and have made the analysis of such systems more tractable.

The use of symbolic manipulation packages, e.g. in MAPLE and MATHEMATICA, which considerably facilitate the calculations will also be discussed.

Lecturer Details:

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