Measure Theory: Exercises 5

1. Show that there is a function f that is not Lebesgue measurable however $|f|$ is Lebesgue measurable.

2. Give an example of a sequence $f_1, f_2, \ldots$ of measurable functions from X of some measure space $(X, \mathcal{A}, \mu)$ to $[-\infty, +\infty]$ and a measurable $f : X \to [-\infty, +\infty]$ such that $\lim_{i\to\infty} f_i(x) = f(x)$ for every $x \in X$, however $\lim_{i\to\infty} \int f_i \, d\mu \neq \int f \, d\mu$.

3. Let $f_1 \geq f_2 \geq \ldots$ be a sequence of measurable functions such that f_1 is integrable. Show that $\int \lim_i f_i \, d\mu = \lim_i \int f_i \, d\mu$.

4. Let $f, g : [0, 1] \to [0, 1]$ be Lebesgue integrable functions such that $\int_I (f - g) \, d\lambda = 0$ for every open interval I. Show that $f = g$ λ-almost everywhere.