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**Lecture 0. 11.10.2010.**

London Taught Course Centre

## **MEASURE-THEORETIC PROBABILITY**

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Course website: My homepage, link to Measure-Theoretic Probability. This also contains past exam papers + solutions.

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- §1. Quadratic variation.
- §2. Itô integral.
- §3. Itô's formula.
- §4. Weak convergence.

## REFERENCES

- [A] D. B. APPLEBAUM (2004): *Lévy processes and stochastic calculus*, CUP.
- [Bach] L. BACHELIER (2006): Louis Bachelier's *Theory of speculation*: The origins of modern finance. Translated and with a commentary by Mark Davis and Alison Etheridge; foreword by Paul A. Samuelson, Princeton UP.
- [Ber] J. BERTOIN (1996): *Lévy processes*. Cambridge Tracts 121, CUP.
- [Bil] P. BILLINGSLEY (1968): *Convergence of probability measures*, Wiley.
- [BF] N. H. BINGHAM and John M. FRY (2010): *Regression: Linear models in statistics*. Springer Undergraduate Mathematics Series (SUMS).
- [Brei] L. BREIMAN (1968), *Probability*, Addison-Wesley.
- [D] J. L. DOOB (1953), *Stochastic processes*, Wiley.
- [F1] W. FELLER (1968): *An introduction to probability theory and its applications*, Vol. 1, 3rd ed., Wiley (1st ed. 1950, 2nd ed. 1957).

- [F2] W. FELLER (1971): *An introduction to probability theory and its applications*, Vol. 2, 2nd ed., Wiley (1st ed. 1966).
- [GS] G. R. GRIMMETT and D. R. STIRZAKER (2001), *Probability and random processes*, 3rd ed., OUP (1st ed. 1982, 2nd ed. 1992).
- [Joh] I. M. JOHNSTONE (2011+): *Function estimation and Gaussian sequence models*, <http://www-stat.stanford.edu/~imj>
- [Kal] O. KALLENBERG (2002): *Foundations of modern probability*, 2nd ed., Springer.
- [K-S] I. KARATZAS and S. E. SHREVE (1988): *Brownian motion and stochastic calculus*, Springer.
- [Kin] J. F. C. KINGMAN (1993): *Poisson processes*, OUP.
- [Kol] A. N. KOLMOGOROV (1933): *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Springer.
- [L-Q] T. J. LYONS and Z. QIAN (2002): *System control and rough paths*, OUP.
- [Mey66] P.-A. MEYER (1966): *Probability and potentials*, Blaisdell.
- [Mey76] P.-A. MEYER (1976): Un cours sur les intégrales stochastiques, *Sém. Probab. X, Lecture Notes in Math.* **511**, 245-400.
- [Mik] T. MIKOSCH (1998): *Elementary stochastic calculus, with finance in view*, World Scientific.
- [Nev] J. NEVEU (1975): *Discrete-parameter martingales*, North-Holland.
- [Ø] B. ØKSENDAL (1998): *Stochastic differential equations. An introduction with applications*, 5th ed., Springer.
- [Pro] P. PROTTER (1990): *Stochastic integration and differential equations. A new approach*, Springer.

[R-Y] D. REVUZ and M. YOR (1999): *Continuous martingales and Brownian motion*, 3rd ed., Springer.

[R-W1] L. C. G. ROGERS and D. WILLIAMS (1994): *Diffusions, Markov processes and martingales, Volume 1: Foundations*, 2nd ed, Wiley.

[R-W2] L. C. G. ROGERS and D. WILLIAMS (1987): *Diffusions, Markov processes and martingales, Volume 2: Itô calculus*, Wiley.

[Ste] J. M. STEELE (2001): *Stochastic calculus and financial applications*, Springer.

[Wil91] D. WILLIAMS (1991): *Probability with martingales*, CUP.

[Wil01] D. WILLIAMS (2001): *Weighing the odds. A course in probability and statistics*, CUP.