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PROBLEMS 1

Q1. Find the areas of (i) triangles, (ii) polygons, (iii) circles, (iv) ellipses.

Q2. Describe how you would go about finding the area of a general region of the plane.

Q3. Would you expect a general region of the plane to *have* an area? If so, how would you go about finding it? If not, why not?

Q4. With $\mathbb{Q}$ the set of rationals, does $I_{\mathbb{Q}}(x)dx$ exist:
(i) as a Riemann integral?
(ii) as a Lebesgue integral?
Give the value of the integral when it exists. Comment on any difference between your answers to (i) and (ii).

Q5 (*Generalised Pythagoras theorem*). A right-angled triangle has sides 1 (the hypotenuse), 2 and 3. A semicircle (or any other plane shape) of area A_1 is drawn with base side 1; similar copies of this are drawn with bases sides 2 and 3, with areas A_2, A_3. Show that

$$A_1 = A_2 + A_3.$$

Deduce Pythagoras' theorem on taking these shapes to be squares.

Recommended Reading.

Not all sets on the line have a length – that is, non-measurable subsets of the line exist. The standard construction of a non-measurable set, due to Vitali, uses the Axiom of Choice (and so is non-constructive!). Refer to a book on Measure Theory, and look up 'non-measurable set' in the index – e.g., Lecture 4 of my Stochastic Processes course (MSc in Mathematical Finance):
Imperial College, London > Mathematics Department > Staff > Staff List > Bingham > Home page > Stochastic Processes;
M. E. MUNROE, *Introduction to measure and integration*, Addison-Wesley, 1953, 142-3;
P. R. HALMOS, *Measure theory*, Springer, 1974 [Van Nostrand, 1950], 69.

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