

LTCC Advanced Course

Title: Spectral Gap in Smooth Dynamics

Basic Details:

- Core Audience: (PhD students in pure math)
- Course Format: (**Extended:** 5 x 2hr lectures)

Course Description:

- Overview:

This course covers the bridge between classical ergodic theory and modern functional analysis. It investigates chaotic, hyperbolic systems—like geodesic flows and Anosov maps—by tracking density evolutions via transfer operators. Utilizing anisotropic Banach spaces and Dolgopyat's inequality, the material demonstrates spectral gaps, exponential mixing, and equidistribution for deterministic and random dynamics.

- Keywords:

Dynamical systems, random walk, spectral gap, anisotropic Banach space

- Syllabus:

1. Ergodic Theory intro. Example: LLN for i.i.d., Weyl's equidistribution theorem, Hyperbolic automorphisms, Subshift of finite type (Finite state Markov chain)
- 2: Expanding maps, Anosov diffeomorphisms, transfer operator, Hennion's theorem
- 3: geodesic flow on negatively curved surface, representation method, symbolic method and Dolgopyat's inequality, Anisotropic Banach space
- 4: Anisotropic Banach space II, spectral gap for random dynamics.
- 5: Application: Equidistribution for random walk

- Recommended reading:

"Introduction to the Modern Theory of Dynamical Systems" by Anatole Katok and Boris Hasselblatt
"Introduction to Dynamical Systems" by Michael Brin and Garrett Stuck
"Ergodic Theory of Random Transformations" by Yuri Kifer

- Prerequisites:

1. Basic knowledge of Functional Analysis, Measure theory and Probability
2. Knowledge in dynamical systems and ergodic theory is preferred

Format:

- No of discussion/problem sheets: 2
- Electronic lecture notes: These will be provided as we go along.
- Necessary support facilities: None.

Lecturer details:

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