LTCC BASIC Course

Title: Introduction to Optimal Transport and its applications

Basic Details:

Core Audience: first year, Pure/App./Stats
Course Format: Extended, 5x 2 hr lectures

Course Description:

Optimal transport is a powerful mathematical theory at the interface of probability theory, partial differential equations, optimization, fluid mechanics and functional analysis. Applications of Optimal Transport are in many areas such as image processing and restoration, statistics, color and texture analysis, and machine learning. In this course I will give an overview of the theory of optimal transport, I will present some efficient algorithms to compute it and I will introduce some of its many applications.

- **Key words:** optimal transport, Wasserstein metric, Brenier map, Kantorovich duality, Sinkhorn algorithm, gradient flow, Wasserstein GAN, optimal assignment, linear programming

- **Syllabus:**

  The lectures will be structured as follows:

  1) Monge and Kantorovich problems (definition, existence of minimizers, Kantorovich duality, equivalence of Monge and Kantorovich formulations)

  2) Characterizations of Optimal Transport Maps and consequences (c-concavity, univariate and square cost cases). Exactly soluble problems

  3) Wasserstein spaces (Topology, Metric, Geodesics, and Riemannian Structure)

  4) Functionals on Wasserstein space (displacement convexity, gradient flows)

  5) Numerical Methods (linear programming; entropic regularization, Sinkhorn algorithm)

Recommended reading:

- Santambrogio, Filippo, *Optimal Transport for Applied Mathematicians,* Calculus of Variations, PDEs, and Modeling, 1-356, 2015

- Villani, Cedric, *Topics in Optimal Transportation* (Graduate Studies in Mathematics), American Mathematical Society, 1-370, 2003
- Ambrosio, Luigi, Gigli, Nicola and Savare, Giuseppe, *Gradient Flows: In Metric Spaces and in the Space of Probability Measures* (Lectures in Mathematics. ETH Zürich), Birkhäuser, 1-343, 2008

Additional or optional reading:

- Friesecke, Gero, *Optimal Transport: A Comprehensive Introduction to Modeling, Analysis, Simulation, Applications*, SIAM, 1-345, 2024

Prerequisites: Basic understanding of measure theory, weak convergence and convex analysis.

Format:
- Number of discussion/problem sheets: 4
- Electronic lecture notes (these are strongly encouraged, as they will form the core of the individual studies of the students): Yes

Lecturer Details:

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