Course Title: Applications of Complex Analysis

Basic Details: Many important problems in applications such as dynamics, fluids and general relativity have solutions given in terms of Abelian functions and Abelian integrals. These are natural generalisations of elliptic functions and elliptic integrals to more general Riemann surfaces. We will introduce these concepts and apply them to solve selected problems.

The second part of the course will consider asymptotic methods that also play a key role in describing solutions of differential equations and are central to areas such as the theory of special functions, random matrix theory and integrable systems.

Core Audience: Mathematics students, both applied and pure.

Course Format: 5 x 2hr lectures

Prerequisities: Basic complex analysis. Elementary knowledge of differential equations. We will look at some applications in elementary mechanics and water waves but no prior knowledge of these areas will be needed.

Course Description:

Keywords: Elliptic functions, elliptic integrals, Abelian functions, theta functions, asymptotics, Stokes phenomenon, exponential asymptotics.

Syllabus:
The content taught each week will be

1. Elliptic functions: Weierstrass and Jacobi elliptic functions. Elliptic integrals. Theta functions.
2. Abelian functions: Riemann surfaces, Abelian integrals, the Riemann theta function.
3. Applications to problems in rigid dynamics (spinning tops) and water waves.
4. Asymptotics of solutions of differential equations in the complex domain.
5. Stokes phenomenon, exponential asymptotics and applications.

Recommended Reading

O. Babelon, D. Bernard and M. Talon, *Introduction to Classical Integrable Systems*, Chp 15

E. Freitag, *Complex Analysis 2*

Format:

Printed lecture notes will be available. Problem sheets will be provided, with full worked solutions made available at the end of the course.

Course lecturer:

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