Empirical Likelihood: Exercise

1. Let $X_1, \cdots, X_n$ be a random sample from a continuous distribution. Construct a statistical test for the null hypothesis that the median of the distribution is 0.

2. Let $X_1, \cdots, X_n$ be a random sample from a distribution with mean 0 and variance σ^2, construct a confidence interval for σ^2.

3. Let $a_1 \leq \cdots \leq a_n$, and $a_1 < 0$ and $a_n > 0$. Show that there exists a unique constant λ between n/a_1 and n/a_n for which $\sum_{j=1}^n \frac{a_j}{n-\lambda a_j} = 0$, and $n > \lambda a_j$ for all $1 \leq j \leq n$.

4. Let $X_1, \cdots, X_n$ is a random sample from a distribution with mean μ, variance σ^2, skewness $\gamma = E\{(X_1 - \mu)^3\}/\sigma^3$ and kurtosis $\kappa = E\{(X_1 - \mu)^4\}/\sigma^4$. By assuming the required regularity conditions, construct an empirical likelihood ratio test for testing the null hypothesis

$$H_0 : \; \mu = 0, \; \sigma^2 = 1, \; \gamma = 0, \; \text{and } \kappa = 3.$$

   State how a bootstrap calibration method can be used to find the critical value of the test.

5. Let $\{(X_i, Y_i), \; 1 \leq i \leq n\}$ be a random sample from a two-dimensional continuous distribution with $P(X > 0) > 0$. Construct an estimation equation for estimating $\theta \equiv \text{Median}(Y_1 | X_1 > 0)$. Construct a statistical test for the hypothesis $H_0 : \theta = 1$.