

LTCC Proposed Course

- Title: Holomorphic Dynamics and Hyperbolic Geometry
- Basic Details:
 - Core Audience: 2nd/3rd year: pure (also of interest to applied)
 - Course Format: extended (10 hours at 2 hours per week)
- Course Description:
 - Keywords: Rational maps, Kleinian groups, Julia sets, limit sets, Mandelbrot set.
 - Syllabus:

Rational maps are self-maps of the Riemann sphere of the form $z \rightarrow p(z)/q(z)$ where $p(z)$ and $q(z)$ are polynomials. Kleinian groups are discrete subgroups of $PSL(2, \mathbf{C})$, acting as conformal automorphisms of the Riemann sphere or isometries of 3-dimensional hyperbolic space. Both theories experienced remarkable advances in the last two decades of the 20th century and are very active areas of continuing research. The aim of the course is to introduce some of the main techniques and results in the two areas, emphasising the strong connections and parallels between them.

 - 1. Dynamics of rational maps:** The Riemann sphere and rational maps (basic essentials from complex analysis); conjugacies, fixed points and periodic orbits (basic essentials from dynamical systems); Fatou and Julia sets, equicontinuity, normal families and Montel's Theorem.
 - 2. Fatou and Julia sets:** Characterisations; classification of types of component of Fatou set; linearization theorems (Siegel, Brjuno, Yoccoz).
 - 3. Hyperbolic geometry and Kleinian groups:** Models of the hyperbolic plane, and 3-space. Isometry groups; Kleinian groups; discontinuity sets and limit sets; fundamental domains for Kleinian groups, Poincaré's polyhedron theorem; examples of Fuchsian and Kleinian groups.
 - 4. Quadratic maps and the Mandelbrot set:** The Mandelbrot set and its connectivity; geography of the Mandelbrot set: internal and external rays; introduction to kneading theory (Milnor-Thurston); open questions.
 - 5. Further topics (selection from the following):** The Measurable Riemann Mapping Theorem and its applications to holomorphic dynamics and Kleinian groups; polynomial-like mappings and renormalisation theory; Thurston's Theorem (characterizing topological branched-covering maps equivalent to rational maps); conformal surgery, matings; the 'Sullivan Dictionary' between holomorphic dynamics and Kleinian groups.
 - Recommended reading:

J. Milnor, Dynamics in One Complex Variable, 3rd edition, Princeton University Press 2006
 - Additional optional reading (more may be suggested during the course):

A. Beardon, Iteration of Rational Functions, Springer Graduate Texts in Mathematics No. 132, 1991
A. Beardon, The Geometry of Discrete Groups, Springer GTM no. 91, 1983
J. Ratcliffe, Foundations of Hyperbolic Manifolds, Springer GTM no. 149, 1994
W. Thurston, Three Dimensional Geometry and Topology, Princeton University Press 1997
D. Mumford, C. Series and D. Wright, Indra's pearls: the vision of Felix Klein, CUP 2002
 - Prerequisites: Undergraduate complex analysis, linear algebra and elementary group theory.
- Format:
 - No of discussion/problem sheets: 4
 - Electronic lecture notes: These will be prepared
 - Necessary support facilities: None (any computing is voluntary, and on student's machine)
 - Necessary software requirements for computing facilities: None

- Proposed timing: 2011-12 or later.

- Lecture/computer session/tutorial/discussion split (hours of each): 10h lectures

- Lecturer Details:

- Lecturer: Shaun Bullett

- Lecturer home institution: QMUL

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