

LTCC Homological algebra mock exam

Let A be an associative unital algebra over a commutative ring k .

1. Let X be a chain complex of A -modules such that $H_n(X) = \{0\}$ for all but finitely many integers n .

(a) Show that there exist a bounded below complex of A -modules Y and a quasi-isomorphism $f : Y \rightarrow X$.

(b) Show that there exist a bounded above complex of A -modules Z and a quasi-isomorphism $g : X \rightarrow Z$.

2. Show that for any A -module, the chain complex

$$\cdots \longrightarrow 0 \longrightarrow U \xrightarrow{\text{Id}_U} U \longrightarrow 0 \longrightarrow \cdots$$

is contractible, where U is in any two consecutive degrees of the chain complex.

3. Let $d : A \rightarrow A$ be a derivation. Show that $d(1) = 0$.

4. For m a positive integer, denote by C_m a cyclic group of order m . For any integer $n \geq 0$, calculate the abelian groups $\text{Ext}_{\mathbb{Z}}^n(C_2, C_2)$ and $\text{Ext}_{\mathbb{Z}}^n(C_5, C_4)$.

5. Suppose that k is a field. Calculate the dimension of $HH^1(k[x]/(x^5))$.
(*Hint:* calculate all derivations on $k[x]/(x^5)$.)

6. Let $(E_r^{p,q})$ be a spectral sequence of A -modules starting at the E_1 -page. Suppose that $E_1^{p,q} = 0$ for all $(p, q) \in \mathbb{Z}^2$ such that $p \neq 0$ and $p \neq 1$. Show that $E_2^{p,q} = E_{\infty}^{p,q}$ for all $(p, q) \in \mathbb{Z}^2$.