LTCC Advanced Course

Course Title: Integrable Systems and Solitons

Basic Details: Integrable systems (equations with many conserved quantities) arise in many applications in pure and applied mathematics, such as fluid mechanics. We will study the inverse scattering transform for soliton equations as well as other solution-generating methods. Other topics will include the Hamilton-Jacobi equation, integrable PDEs of hydrodynamic type and soliton gases.

Core Audience: Applied mathematics students

Course Format: 5 x 2hr lectures

Prerequisites: Basic complex analysis, especially contour integrals. Basic partial differential equations. Some familiarity with Hamiltonian systems and asymptotic methods is desirable but not necessary.

Course Description:

Keywords: Integrable systems, solitons, Lax pairs, Bäcklund transformations, equations of hydrodynamic type.

Syllabus:
The content taught each week will be

1. Introductory material. Hamiltonian systems. Liouville integrability. The Hamilton-Jacobi equation.
2. The Korteweg-de Vries (KdV) equation. Lax pairs. Bäcklund transformations. Hirota's method. Solitons. The direct scattering problem for the KdV equation.
3. The inverse scattering transform.
4. Soliton gases.
5. Integrable equations of hydrodynamic type.

Recommended Reading:

Ablowitz and Clarkson. Solitons, nonlinear evolution equations and inverse scattering.
Drazin and Johnson. Solitons: an introduction.

Format:

Printed lecture notes will be available. Problem sheets will be provided, with full worked solutions made available at the end of the course.

Lecturer details:
Course lecturer: Professor Rod Halburd (UCL).
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