

Analytical Methods exam paper 2026.

Consider the following partial differential equation for $u(x, y)$:

$$\frac{1}{4} \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + \varepsilon x^2 u = -1,$$

one of whose boundary conditions is

$$\frac{\partial u}{\partial y}(x, 0) = 0, \quad \text{for all } x,$$

in which ε is a small positive parameter.

Answer **either one** of the following questions (your choice):

1. Seek a solution which is independent of y . Identify all possible scale-and-stretch balances and sketch them in a diagram. Identify the distinguished scaling which results in a parameter-free ODE for a function $F(z)$, where you should define both F and z in terms of u , x and ε . Calculate a power-series solution for $F(z)$ up to order z^6 .

Now impose additionally the boundary conditions

$$u(1, y) = 2, \quad \frac{\partial u}{\partial x}(1, y) = 0.$$

Calculate the first two terms of a regular perturbation expansion for u . What is the region in x in which your solution is valid?

2. Taking boundary conditions

$$\frac{\partial u}{\partial y}(x, 0) = 0, \quad u(x, 0) = 0,$$

find the first two terms (i.e. order 1 and order ε) in a regular perturbation expansion for $u(x, y)$.

You should **not** attempt more than one question: one is enough!