

LTCC Exam 2026 – Mathematical Biology

1 Section 1: Random Walkers

Open the random walker code that we studied in class by going to: <https://github.com/philip-pearce/randomwalkers>

1. Plot the mean-squared distance vs. time for the random walkers in the discrete code.
2. Edit the discrete code so that the random walkers are able to stay in the same place at each time step, with equal likelihood as moving right or moving left (i.e. 3 outcomes at each step, with probability 1/3 each). Run a simulation of the discrete code and plot the MSD vs. time
3. Plot the histogram of particle positions in the discrete code at various timepoints from the results of the simulation in part 2. Write out a derivation of the diffusion equation, starting from your new rules for discrete walkers above (adapt the method in the lecture notes). Edit the method of lines method in the code, so that you can run a continuum simulation of the diffusion equation with parameters corresponding to your new rules for the walkers. Plot the concentration of random walkers at various time points (this can be on the same graph as your histogram from part a, or on a different graph). Do your discrete and continuum results agree?
4. Edit the discrete code so that the random walkers are more likely to move right than left (e.g. 2/3 probability of moving right, 1/3 probability of moving left). Derive the continuum equation in this case – which term explains the new dynamics?

2 Section 2: Michaelis-Menten Enzyme Kinetics

Open the Michaelis-Menten resources and code that we studied in class by going to: https://github.com/Edwina-Yeo/LTCC-Math-Bio/MM_matched_asymptotics.ipynb

1. Equations (1-4) represent the dimensional reactions between the enzyme, E , substrate S , product P and the complex C .
 - (a) By adding together combinations of equations (1-4), derive 2 conservation laws for the system.
 - (b) What do these conservation laws mean physically?
2. The system of equations (1-4) is nondimensionalised by defining a new time variable $\tau = k_1 S_0 t$ and scaled concentration variables $c = C/E_0$ and $s = S/S_0$, $P = S_0 p$, $E = E_0 e$. What do the non-dimensionalised rate parameters $\lambda = k_3/(k_1 S_0)$, $\mu = (k_2 + k_3)/(k_1 S_0)$ mean physically?
3. In this nondimensionalisation a parameter $\epsilon = E_0/S_0$, which physically represents the relative concentrations of enzyme to substrate initially is defined to be small. This allows us to carry out asymptotic analysis of the system using the method of matched asymptotic expansions. The code in the attached notebook solves the ODEs numerically for $t \in [0, 6]$ and also plots the composite solution to the system we derived in class asymptotically.

Define the mean error made between the numerical ODE substrate solution $s(t, \epsilon)$ and the asymptotic composite substrate solution $\tilde{s}(t, \epsilon)$ as follows

$$\text{Error}(\epsilon) = \frac{1}{N} \sum_{i=0}^N |s(t_i, \epsilon) - \tilde{s}(t_i, \epsilon)| \quad (1)$$

where $t_i = 6/Ni$ are N points in the interval $t \in [0, 6]$. For $N = 100$, make a plot of ϵ vs $\text{Error}(\epsilon)$ for $\epsilon \in [10^{-4}, 10]$. At what value of ϵ does the error first increase above 1% of the maximum value of the ODE numerical solution (defined mathematically as $\text{Error}(\epsilon) > 0.01$)? Can you numerically estimate how the error scales with ϵ , why do you think this is?

Please send the answers in PDF format to office@ltcc.ac.uk by the end of the day on 9 April if you are taking 1 or 2 LTCC exams or by the end of the day on 13 April if you are sitting 3 or more exams.