Laplacian eigenvalues and optimality LTCC Intensive Course

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Eigenvalues of the Laplacian matrix of a graph have been widely used in studying connectivity and expansion properties of networks.

Independently, statisticians introduced various *optimality criteria* in experimental design, the goal being to obtain more accurate estimates of quantities of interest in an experiment. It turns out that the most popular of these optimality criteria for block designs are determined by the Laplacian eigenvalues of the *concurrence graph*, or the *Levi graph*, of the design.

The most important optimality criteria, called A (average), D (determinant) and E (extreme), are related to the conductance of the graph as an electrical network, the number of spanning trees, and the isoperimetric properties of the graphs.

The number of spanning trees is also an evaluation of the Tutte polynomial of the graph, and is the subject of the Merino–Welsh conjecture relating it to acyclic and totally cyclic orientations, of interest in their own right.

References

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[2] B. Bollobás, *Modern Graph Theory*. Graduate Texts in Mathematics **184**, Springer, New York, 1998, Chapters II and IX.