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**Lecture 0. 30.9.2013.**

London Taught Course Centre

## **MEASURE-THEORETIC PROBABILITY**

Professor N. H. BINGHAM, Autumn 2013

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Course website: My homepage, link to Measure-Theoretic Probability. This also contains past exam papers + solutions.

This 10-hour course [5 weeks, 2 hours each] can cover only the bare essentials of a vast subject, and of course there is very much more to say. For a 20-hour version see the Stochastic Analysis link on my homepage [10 x 2 hours, LSE]; for a 30-hour version see the Stochastic Processes link on my homepage [30 x 1 hour, Imperial]. For a range of useful books, see the References after the Table of Contents below.

### **Contents**

Week 1. 30.9.2013. Chapter I: Probability background.

§1. Measure.

§2. Integral.

§3. Probability.

§4. Equivalent measures and the Radon-Nikodym theorem.

Week 2. 7.10.2013. Chapter II: Conditioning. Stochastic Processes.

§1. Conditional expectation.

§2. Properties of conditional expectation.

§3. Filtrations.

§4. Discrete-parameter stochastic processes.

§5. Stochastic processes in continuous time.

§6. Renewal processes; Poisson process.

Week 3. 14.10.2013. Chapter III: Martingales.

- §1. Discrete-parameter martingales.
- §2. Martingale convergence.
- §3. Uniformly integrable martingales.
- §4. Stopping times and optional stopping.
- §5. Doob decomposition.
- §6. Examples.
- §7. Continuous-parameter martingales.
- §8. Poisson processes; Lévy processes

Week 4. 21.10.2013. Chapter IV: Stochastic processes in continuous time.  
Brownian motion.

- §1. Markov processes.
- §2. Gaussian processes.
- §3. Brownian motion.

Week 5. 28.10.2013. Itô (stochastic) calculus. Weak convergence.

- §1. Quadratic variation.
- §2. Itô integral.
- §3. Itô's formula.
- §4. Weak convergence.

## REFERENCES

[A] D. B. APPLEBAUM (2004): *Lévy processes and stochastic calculus*, CUP.

[Bach] L. BACHELIER (2006): Louis Bachelier's *Theory of speculation*: The origins of modern finance. Translated and with a commentary by Mark Davis and Alison Etheridge; foreword by Paul A. Samuelson, Princeton UP.

[Ber] J. BERTOIN (1996): *Lévy processes*. Cambridge Tracts 121, CUP.

[Bil] P. BILLINGSLEY (1968): *Convergence of probability measures*, Wiley.

[BF] N. H. BINGHAM and John M. FRY: *Regression: Linear models in statistics*. Springer Undergraduate Mathematics Series (SUMS), 2010.

[BK] N. H. BINGHAM and Rüdiger KIESEL: *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed. Springer Finance, 2004 (1st ed. 1998).

[Brei] L. BREIMAN (1968), *Probability*, Addison-Wesley.

[D] J. L. DOOB (1953), *Stochastic processes*, Wiley.

[F1] W. FELLER (1968): *An introduction to probability theory and its applications*, Vol. 1, 3rd ed., Wiley (1st ed. 1950, 2nd ed. 1957).

[F2] W. FELLER (1971): *An introduction to probability theory and its applications*, Vol. 2, 2nd ed., Wiley (1st ed. 1966).

[GS] G. R. GRIMMETT and D. R. STIRZAKER (2001), *Probability and random processes*, 3rd ed., OUP (1st ed. 1982, 2nd ed. 1992).

[Joh] I. M. JOHNSTONE (2011+): *Function estimation and Gaussian sequence models*, <http://www-stat.stanford.edu/~imj>

[Kal] O. KALLENBERG (2002): *Foundations of modern probability*, 2nd ed., Springer.

[K-S] I. KARATZAS and S. E. SHREVE (1988): *Brownian motion and stochastic calculus*, Springer.

[Kin] J. F. C. KINGMAN (1993): *Poisson processes*, OUP.

[Kol] A. N. KOLMOGOROV (1933): *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Springer.

[L-Q] T. J. LYONS and Z. QIAN (2002): *System control and rough paths*, OUP.

[Mey66] P.-A. MEYER (1966): *Probability and potentials*, Blaisdell.

[Mey76] P.-A. MEYER (1976): Un cours sur les intégrales stochastiques, *Sém. Probab. X, Lecture Notes in Math.* **511**, 245-400.

- [Mik] T. MIKOSCH (1998): *Elementary stochastic calculus, with finance in view*, World Scientific.
- [Nev] J. NEVEU (1975): *Discrete-parameter martingales*, North-Holland.
- [Ø] B. ØKSENDAL (1998): *Stochastic differential equations. An introduction with applications*, 5th ed., Springer.
- [Pro] P. PROTTER (1990): *Stochastic integration and differential equations. A new approach*, Springer.
- [R-Y] D. REVUZ and M. YOR (1999): *Continuous martingales and Brownian motion*, 3rd ed., Springer.
- [R-W1] L. C. G. ROGERS and D. WILLIAMS (1994): *Diffusions, Markov processes and martingales, Volume 1: Foundations*, 2nd ed, Wiley.
- [R-W2] L. C. G. ROGERS and D. WILLIAMS (1987): *Diffusions, Markov processes and martingales, Volume 2: Itô calculus*, Wiley.
- [Ste] J. M. STEELE (2001): *Stochastic calculus and financial applications*, Springer.
- [Wil91] D. WILLIAMS (1991): *Probability with martingales*, CUP.
- [Wil01] D. WILLIAMS (2001): *Weighing the odds. A course in probability and statistics*, CUP.